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INDIA'S NUMBER 1 MATHEMATICS MONTHLY

OCTOBER 1996 Rs.12

SOLVED PAPER

B.I.T. RANCHI '96

IIT-JEE '97 PRACTICE TEST SERIES

PROBLEMS OF THE MONTH

VECTOR

INTERNATIONAL MATH OLYMPIAD PROBLEMS

EXAMINER'S MIND

CONIC SECTION

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Editor Mahabir Singh

Associate Editors Anil Ahlawat & Y.P.S. Tomar

Circulation Officer S.K. Mishra

Composing K Rajeev Nair

Consultants Centre for Marketing Ideas, C-17 Anand Niketan, New Delhi - 110021

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* Ajay C. Ramadoss has won a gold medal at International Mathematics Olympiad held in Mumbai.

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BIT RANCHI '96 SOLVED PAPER

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1. (a) Let $f(x) = [1 + \tan x][1 + \tan(\pi/4 - x)]$ and let $g(x)$ be defined for every real number x . Prove that the composite function $g[f(x)]$ is constant for all those x for which $f(x)$ is defined.

Soln.

$$\begin{aligned}\text{Since } f(x) &= [1 + \tan x][1 + \tan(\pi/4 - x)] \\ &= [1 + \tan x] \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] \\ &= [1 + \tan x] \left[\frac{2}{1 + \tan x} \right] = 2\end{aligned}$$

provided $1 + \tan x \neq 0$ i.e. $\tan x \neq -1$ i.e. $x \neq n\pi - \pi/4$
 $\therefore g[f(x)] = g(2) = \text{constant}$

- (b) Let C denote the circle $x^2 + y^2 - 6y + 5 = 0$. Determine the equation of a circle (say D) which is concentric with C and the angle between the tangents to D , from every point on the circumference of C is a given constant.

Soln.

$$\text{Since } C \equiv x^2 + y^2 - 6y + 5 = 0$$

centre $O \equiv (0, 3)$,

$$\text{radius } PO = \sqrt{9 - 5} = 2$$

Let angle between the tangents to D from every point on the circumference of C is 2θ .

$$\therefore \angle APO = \theta, \angle PAO = 90^\circ$$

$$\sin \theta = \frac{AO}{OP} \therefore AO = 2 \sin \theta$$

Since both are concentric, so O is also centre of circle D .

$\therefore$ Equation of circle D is

$$(x - 0)^2 + (y - 3)^2 = AO^2 \Rightarrow x^2 + y^2 - 6y + 9 = 4 \sin^2 \theta$$

$$\therefore x^2 + y^2 - 6y + 9 - 4 \sin^2 \theta = 0$$

This is required equation of circle D .

2. Let $[a]$ denote the largest integer not exceeding the real number a . If x and y satisfy the equations

$y = 2[x] + 3$, and $y = 3[x - 2] + 5$, simultaneously, determine $[x + y]$

Soln.

If $x = a + f$ where a is the integral part and f is the positive fractional part.

$$\therefore y = 2[x] + 3 = 2[a + f] + 3 = 2a + 3 \quad \dots\dots\dots (i)$$

$$y = 3[x - 2] + 5 = 3[a + f - 2] + 5 = 3(a - 2) + 5 = 3a - 1 \quad \dots\dots\dots (ii)$$

Subtract eqn. (i) from eqn. (ii), we get,

$$0 = a - 4$$

$$\therefore a = 4$$

$$\therefore y = 3 \times 4 - 1 = 11$$

$$[x + y] = [a + f + y] = [4 + f + 11] = [15 + f] = 15$$

$$\therefore [x + y] = 15.$$

3. If x, y and z are real numbers such that $x + y + z = 5$ and $xy + yz + zx = 3$, determine the largest possible value of x .

Soln.

$$\text{Since } x + y + z = 5 \Rightarrow y = 5 - x - z$$

$$\text{Again, } xy + yz + zx = 3$$

$$\Rightarrow x(5 - x - z) + (5 - x - z)z + zx = 3$$

$$\Rightarrow 5x - x^2 - xz + 5z - xz - z^2 + zx = 3$$

$$\Rightarrow z^2 + (x - 5)z + (x^2 - 5x + 3) = 0$$

$$\therefore z \text{ is real so discriminant } b^2 - 4ac \geq 0$$

$$\therefore (x - 5)^2 + 4(x^2 - 5x + 3) \geq 0$$

$$\Rightarrow x^2 - 10x + 25 - 4x^2 + 20x - 12 \geq 0$$

$$\Rightarrow 3x^2 - 10x - 13 \leq 0 \Rightarrow 3x^2 - 13x + 3x - 13 \leq 0$$

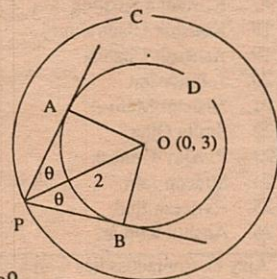
$$\Rightarrow 3x \left(x - \frac{13}{3} \right) + 3 \left(x - \frac{13}{3} \right) \leq 0$$

$$\Rightarrow (3x + 3) \left(x - \frac{13}{3} \right) \leq 0$$

$$\Rightarrow (x + 1) \left(x - \frac{13}{3} \right) \leq 0$$

$$\therefore -1 \leq x \leq \frac{13}{3}$$

$$\therefore \text{Largest value of } x \text{ is } 13/3.$$



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4. The quadratic equations $x^2 + ax + 3b = 0$ and $x^2 + bx + 3a = 0$ have only one common root. If a and b are non-zero real numbers, prove that the other roots of these equations satisfy the equation $x^2 + 3x + ab = 0$

Soln.

Given equations are

$$x^2 + ax + 3b = 0 \text{ and } x^2 + bx + 3a = 0$$

Let α is common root then

$$\alpha^2 + a\alpha + 3b = 0 \text{ and } \alpha^2 + b\alpha + 3a = 0$$

$$\frac{\alpha^2}{3a^2 - 3b^2} = \frac{\alpha}{3b - 3a} = \frac{1}{b - a}$$

$$\Rightarrow \alpha = \frac{3a^2 - 3b^2}{3b - 3a} = -(a + b) \quad \dots\dots(i)$$

$$\text{Also } \alpha = \frac{3(b - a)}{b - a} = 3 \quad \dots\dots(ii)$$

$$\therefore -(a + b) = 3 \quad \dots\dots(iii)$$

Let β_1 and β_2 be the other roots of the equations.

$$\therefore \alpha\beta_1 = 3b \Rightarrow \beta_1 = b$$

$$\therefore \alpha\beta_2 = 3a \therefore \beta_2 = a$$

$$\text{so, } \beta_1 + \beta_2 = a + b = -3 \Rightarrow \beta_1\beta_2 = ab$$

Required equation

$$\Rightarrow x^2 - (\beta_1 + \beta_2)x + \beta_1\beta_2 = 0$$

$$\Rightarrow x^2 + 3x + ab = 0$$

where β_1 & β_2 are two roots of this equations.

5. If $\log_2 x + \log_2 y \geq 6$, prove that the smallest possible value of $x + y$ is 16

Soln.

$$\text{Since } \log_2 x + \log_2 y \geq 6$$

$$\Rightarrow \log_2 (xy) \geq 6 \Rightarrow xy \geq 2^6$$

$$\therefore x + y = \sqrt{(x - y)^2 + 4xy} \geq \sqrt{4xy} \geq \sqrt{4 \cdot 26} \geq 16$$

so smallest value of $x + y$ is 16.

6. There are three non-zero complex numbers which satisfy the equation $i\bar{z} = z^2$

Soln.

$$\text{Let } z = x + iy \text{ then } \bar{z} = x - iy$$

$$\text{From question, } i\bar{z} = z^2$$

$$i(x - iy) = (x + iy)^2$$

$$\Rightarrow i(x - iy) = x^2 - y^2 + i2xy$$

$$\Rightarrow ix + y = x^2 - y^2 + i2xy$$

Equating real and imaginary parts, we get

$$x^2 - y^2 = y \quad \dots\dots(i) \text{ and } 2xy = x \quad \dots\dots(ii)$$

$$\Rightarrow (2xy - x) = 0 \Rightarrow x(2y - 1) = 0$$

$$\text{Either } x = 0 \text{ or } y = \frac{1}{2}$$

Put $x = 0$ in the eqn. (i) we get

$$-y^2 = y \Rightarrow y^2 + y = 0 \Rightarrow y(y + 1) = 0$$

$$\text{Either } y = 0 \text{ or } y = -1$$

Again put $y = \frac{1}{2}$ in the equation (i) we get

$$x^2 - (\frac{1}{2})^2 = \frac{1}{2} \Rightarrow x^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\therefore x = \pm \frac{\sqrt{3}}{2}$$

$$\therefore z = 0 - i1, + \frac{\sqrt{3}}{2} + i\frac{1}{2}, -\frac{\sqrt{3}}{2} + i\frac{1}{2}$$

so three non zero complex numbers are = 1

$$-i, \frac{\sqrt{3}}{2} + i\frac{1}{2}, -\frac{\sqrt{3}}{2} + i\frac{1}{2}$$

7. Choose any 9 distinct integers. Those 9 integers can be arranged to give 9! determinants, each of order 3 by 3.

Determine the sum of these 9! determinants.

Soln.

Let those 9 numbers are a, b, c, d, e, f, g, h and i

$$\text{so one determinant is } \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

a can be arranged in 9 ways

b can be arranged in 8 ways

c can be arranged in 7 ways

d can be arranged in 6 ways

e can be arranged in 5 ways

f can be arranged in 4 ways

g can be arranged in 3 ways

h can be arranged in 2 ways

i can be arranged in 1 way

so 9 integers can be arranged to form 9! determinants.

Sum of the 9! determinants

$$= \begin{vmatrix} \sum a & \sum a & \sum a \\ \sum a & \sum a & \sum a \\ \sum a & \sum a & \sum a \end{vmatrix} = (\sum a)^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

8. Determine all those value of p for which two chords can be drawn from the point (P, P) on the circle

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Sridhar Mechinaju	Kalpakdam	49
J. Praveena	Tiruchirappalli	53
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Purvi Gupta	Jaipur	68
Hansh Nanda	Amritsar	73
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Neel Kishore Verma	Allahabad	91
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Nishith Kumar Singh	Durg	150
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$(x - P)^2 + y^2 = P^2$ both of which are bisected by the straight line $x - 2y + 2 = 0$

Soln.

Let AB and AC are two chords drawn from a point A(P, P).

Let the line

$x - 2y + 2 = 0$ bisects the line AB on the point P.

Let $P = \left[\alpha, \frac{1}{2}(\alpha + 2) \right]$ and $B = [x, y]$

Since P is middle point of AB

$$\therefore \alpha = \frac{P+x}{2} \Rightarrow x = 2\alpha - P.$$

$$\text{Also } \frac{1}{2}(\alpha + 2) = \frac{y+P}{2} \Rightarrow y = \alpha + 2 - P$$

$\therefore B = (x, y)$ is a point on the circle $(x - P)^2 + y^2 = P^2$

$$\therefore (x - P)^2 + y^2 = P^2$$

$$\Rightarrow [2\alpha - P - P]^2 + [\alpha + 2 - P]^2 = P^2$$

$$\Rightarrow 4\alpha^2 - 8\alpha P + 4P^2 + \alpha^2 + 4 + P^2 + 4\alpha - 4P - 2\alpha P = P^2$$

$$\Rightarrow 5\alpha^2 + (4 - 10P)\alpha + (4P^2 - 4P + 4) = 0$$

for α real and different we get,

$$(4 - 10P)^2 - 4 \cdot 5(4P^2 - 4P + 4) > 0$$

$$\Rightarrow 16 - 80P + 100P^2 - 80P^2 + 80P - 80 > 0$$

$$\Rightarrow 5P^2 - 16 > 0 \Rightarrow P^2 > \frac{16}{5}$$

$$\therefore P > \frac{4}{\sqrt{5}}, P < -\frac{4}{\sqrt{5}}$$

$$P \in \left[-\alpha, -\frac{4}{\sqrt{5}} \right] \cup \left[\frac{4}{\sqrt{5}}, \alpha \right].$$

9. The angles A, B and C of a triangle ABC are in arithmetic progression. If $2b^2 = 3c^2$, determine the angle A.

Soln.

Since A, B and C are in A.P.

$$\text{so } 2B = A + C$$

$$\text{we know, } A + B + C = \pi \Rightarrow 3B = \pi$$

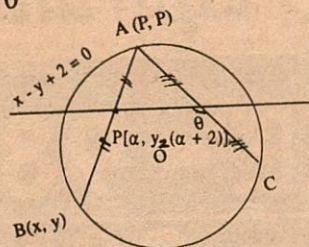
$$\therefore B = \frac{\pi}{3}$$

$$\therefore 2b^2 = 3c^2$$

$$\Rightarrow \frac{b}{c} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{\sin \pi/3}{\sin \pi/4} \Rightarrow \frac{\sin \pi/3}{\sin c} = \frac{\sin \pi/3}{\sin \pi/4}$$

$$\text{so, } c = \pi/4$$

$$\therefore A = \pi - B - C = \pi - \pi/3 - \pi/4$$



$$\angle A = \frac{5\pi}{12} \text{ radians.}$$

10. Prove that

$$\cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7} = \frac{1}{2}$$

Soln.

$$\therefore \cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7}$$

$$= \frac{1}{2 \sin \pi/7} \left[2 \sin \frac{\pi}{7} \cdot \cos \frac{\pi}{7} + 2 \sin \frac{\pi}{7} \cdot \cos \frac{3\pi}{7} + 2 \sin \frac{\pi}{7} \cdot \cos \frac{5\pi}{7} \right]$$

$$= \frac{1}{2 \sin \pi/7} \left[\sin \frac{2\pi}{7} + \sin \frac{4\pi}{7} - \sin \frac{2\pi}{7} + \sin \frac{6\pi}{7} - \sin \frac{4\pi}{7} \right]$$

$$= \frac{1}{2 \sin \pi/7} \times \sin \left(\pi - \frac{\pi}{7} \right) = \frac{1}{2}$$

$$\therefore \cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7} = \frac{1}{2}$$

11. Use the formula

$$(\vec{P} \times \vec{Q}) \times \vec{R} = (\vec{P} \cdot \vec{R}) \vec{Q} - (\vec{Q} \cdot \vec{R}) \vec{P}$$

to prove that

$$\{[(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} \times \vec{C})\} \cdot (\vec{B} + \vec{C}) = 0$$

where $|\vec{B}| = |\vec{C}| = 1$

Soln.

$$\therefore \{[(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} \times \vec{C})\} \cdot (\vec{B} + \vec{C})$$

$$= \{[(\vec{A} + \vec{B}) \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{C}) - [(\vec{A} + \vec{C}) \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{B})\} \cdot (\vec{B} + \vec{C})$$

$$= \{[(\vec{A} + \vec{B}) \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{C}) - [(\vec{A} + \vec{C}) \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{B})\} \cdot (\vec{B} + \vec{C})$$

$$= \{[(\vec{A} \cdot \vec{B} \times \vec{C}) (\vec{A} + \vec{C}) - [(\vec{A} \cdot \vec{B} \times \vec{C}) (\vec{A} + \vec{B})] \cdot (\vec{B} + \vec{C})\}$$

$$\therefore \vec{B} \cdot \vec{B} \times \vec{C} = \vec{C} \cdot \vec{B} \times \vec{C} = 0$$

$$= [(\vec{A} \cdot \vec{B} \times \vec{C}) (\vec{A} + \vec{C}) - (\vec{A} \cdot \vec{B} \times \vec{C}) (\vec{A} + \vec{B})] \cdot (\vec{B} + \vec{C})$$

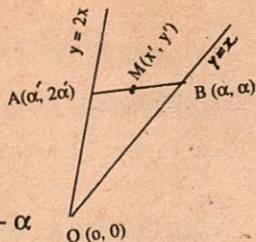
$$= [(\vec{A} \cdot \vec{B} \times \vec{C}) (\vec{C}^2 - \vec{B}^2)] \cdot (\vec{B} + \vec{C}) = [(\vec{A} \cdot \vec{B} \times \vec{C}) (1 - 1)] = 0$$

$$\therefore \{[(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} \times \vec{C})\} \cdot (\vec{B} + \vec{C}) = 0$$

12. Let AB be a line segment of length 4 with A on the line $y = 2x$ and B on the line $y = x$. Determine the locus of the middle points of all such line segments.

Soln.

Let $A \equiv (\alpha', 2\alpha')$
and $B \equiv (\alpha, \alpha)$
mid point of AB,
 $M \equiv (x', y')$



$$\therefore x' = \frac{\alpha' + \alpha}{2} \Rightarrow \alpha' = 2x' - \alpha$$

$$\text{Again } y' = \frac{2\alpha' + \alpha}{2} \Rightarrow \alpha' = \frac{(2y' - \alpha)}{2}$$

$$\therefore 2x' - \alpha = \frac{(2y' - \alpha)}{2}$$

$$\Rightarrow 4x' - 2y' = \alpha \quad \dots (i)$$

$$\therefore MB^2 = 4$$

$$\Rightarrow (x' - \alpha)^2 + (y' - \alpha)^2 = 4$$

$$\Rightarrow (x' - 4x' + 2y')^2 + (y' - 4x' + 2y')^2 = 4$$

$$\Rightarrow 9x'^2 - 12x'y' + 4y'^2 + 9y'^2 + 16x'^2 - 24x'y' = 4$$

$$\Rightarrow 25x'^2 + 13y'^2 - 36x'y' = 4$$

$\therefore$ Locus of middle point of AB is
 $25x^2 + 13y^2 - 36xy = 4$.

13. Let $P(x)$ denote the probability of the occurrence of event x . Plot all those points $(x, y) = (P(A), P(B))$ in a plane which satisfy the conditions
Soln.

$$\therefore P(A \cup B) \geq \frac{3}{4} \Rightarrow P(A) + P(B) - P(A \cap B) \geq \frac{3}{4}$$

$$\Rightarrow P(A) + P(B) \geq \frac{3}{4} + P(A \cap B)$$

$$\Rightarrow x + y \geq \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

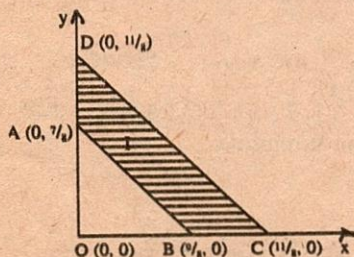
$$\text{so, } x + y \geq \frac{7}{8} \quad \dots (i)$$

Again we know, $P(A \cup B) \leq 1$

$$\Rightarrow P(A) + P(B) - P(A \cap B) \leq 1$$

$$\Rightarrow P(A) + P(B) \leq 1 + P(A \cap B)$$

$$\Rightarrow P(A) + P(B) \leq 1 + \frac{3}{8} \Rightarrow x + y \leq \frac{11}{8} \quad \dots (ii)$$



Required plot graphical part I i.e. area ABCD

14. The coefficients a , b and c of the quadratic equation $ax^2 + bx + c = 0$ are determined by throwing a dice three times. Determine the probability that the roots of the equation are real.

Soln.

When a dice is thrown, possible outcomes are 1, 2, 3, 4, 5, 6.

Hence a , b & c each can have six values 1, 2, 3, 4, 5, 6.

Hence $n(s) = 6 \times 6 \times 6$.

Let E be the event of getting real roots so that

$$D = b^2 - 4ac \geq 0$$

Then

$$E = \{b = 2, \{(a, c)\} = \{(1, 1)\} \dots \dots \dots 1$$

$$b = 3, \{(a, c)\} = \{(1, 1), (1, 2), (2, 1)\} \dots \dots \dots 3$$

$$b = 4, \{(a, c)\} = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (4, 1)\} \dots \dots \dots 8$$

$$b = 5, \{(a, c)\} = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1), (5, 1), (6, 1)\} \dots \dots \dots 14$$

$$b = 6, \{(a, c)\} = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1), (6, 1)\} \dots \dots \dots 17$$

$$n(E) = 1 + 3 + 8 + 14 + 17 = 43$$

$$P(E) = \frac{n(E)}{n(s)} = \frac{43}{6 \times 6 \times 6} = \frac{43}{216}$$

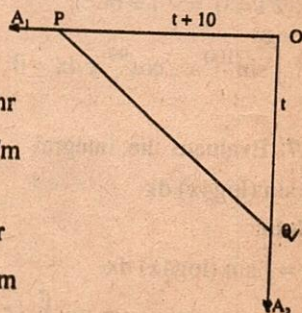
15. An airplane flew over a small airport at 300 kms/hr. Ten minutes later, another airplane flew over the airport at 240 kms/hr. at the same altitude. If the first airplane was flying west and other the second was flying south and both the planes maintained their respective speeds find how fast they were separating 20 minutes after the second plane flew over the airport.

Soln.

$$\begin{aligned} \text{velocity of } A_1 &= v_1 \\ &= 300 \text{ km/hr} \\ &= \frac{300}{60} \text{ km/m} = 5 \text{ km/m} \end{aligned}$$

$$\begin{aligned} \text{velocity of } A_2 &= v_2 \\ &= 240 \text{ km/hr} \\ &= \frac{240}{60} \text{ km/m} = 4 \text{ km/m} \end{aligned}$$

Let after t minutes A_2 reached on the point Q and after $t + 10$ minutes A_1 reached on the point P .



Where A_1 & A_2 are first and second airplane respectively.

$$\therefore OP = 5(t + 10) = 5t + 50 \text{ kms}$$

$$OQ = 4t \text{ kms}$$

$$PQ^2 = OP^2 + OQ^2$$

$$= (5t + 50)^2 + (4t)^2$$

$$= 25t^2 + 500t + 2500 + 16t^2$$

$$= 41t^2 + 500t + 2500$$

$$\therefore PQ = \sqrt{41t^2 + 500t + 2500}$$

$$\frac{d}{dt}(PQ) = \frac{1}{2\sqrt{41t^2 + 500t + 2500}} \times [82t + 500]$$

After 20 minutes i.e. $t = 20$

$$= \frac{1640 + 500}{2\sqrt{820 \times 20 + 500 \times 20 + 2500}}$$

$$= \frac{2140}{2\sqrt{16400 + 10000 + 2500}} = \frac{2140}{2\sqrt{28900}}$$

$$= \frac{2140}{2 \times 170} = 6.3 \text{ kms. (approx.)}$$

16. Use the properties of the definite integrals to evaluate the integral

$$\int_0^\pi \sin^{100} x \cdot \cos^{99} x \, dx$$

Soln.

$$I = \int_0^\pi \sin^{100} x \cdot \cos^{99} x \, dx$$

$$= \int_0^\pi \sin^{100}(\pi - x) \cdot \cos^{99}(\pi - x) \, dx$$

$$= - \int_0^\pi \sin^{100} x \cdot \cos^{99} x \, dx = -I$$

$$\therefore 2I = 0 \Rightarrow I = 0$$

$$\therefore \int_0^\pi \sin^{100} x \cdot \cos^{99} x \, dx = 0$$

17. Evaluate the integral

$$\int \sin(\log 4x) \, dx$$

Soln.

$$I = \int \sin(\log 4x) \, dx$$

$$dx = \sin(\log 4x) \int dx - \int \left\{ \frac{d}{dx} \sin(\log 4x) \right\} \int dx$$

$$= \sin(\log 4x) \cdot x - \int x \cdot \frac{\cos(\log 4x)}{x} \, dx$$

$$= \sin(\log 4x) \cdot x - \cos(\log 4x) \int dx +$$

$$\int \left\{ \int dx \left(\frac{d}{dx} \cos(\log 4x) \right) \right\} dx$$

$$= \sin(\log 4x) \cdot x - \cos(\log 4x) \cdot x -$$

$$\int x \cdot \frac{\sin(\log 4x)}{x} \, dx$$

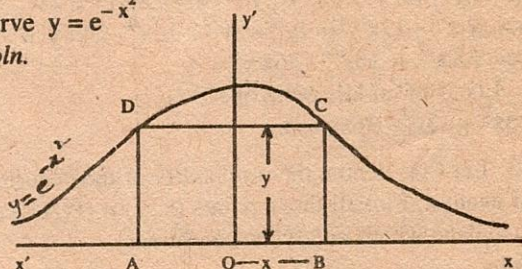
$$= x[\sin(\log 4x) - \cos(\log 4x)] - I$$

$$\therefore I = \frac{x}{2} [\sin(\log 4x) - \cos(\log 4x)].$$

18. Determine the largest area of the rectangle whose base is on the x-axis and two of its vertices lie on the

curve $y = e^{-x^2}$

Soln.



Let ABCD is a rectangle whose base is on the x-axis and two vertices A and B lie on the curve $y = e^{-x^2}$ we have $OA = OB$

Let $OA = OB = x$ and $AD = y$

$$\text{Area of rectangle} = 2xy \Rightarrow A = 2xe^{-x^2}$$

$$\frac{d(A)}{dx} = 2 \frac{d}{dx} [x \cdot e^{-x^2}]$$

$$= 2 \left[e^{-x^2} + x \cdot \frac{d}{dx} e^{-x^2} \cdot \frac{dx^2}{dx} \right]$$

$$= 2[e^{-x^2} - 2x^2 e^{-x^2}] = 2e^{-x^2} [1 - 2x^2]$$

$$\frac{d^2 A}{dx^2} = 2 \left[e^{-x^2} (0 - 4x) + (1 - 2x^2) \frac{d}{dx} e^{-x^2} \times \frac{dx^2}{dx} \right]$$

$$= 2[e^{-x^2} (0 - 4x) - (1 - 2x^2) (2xe^{-x^2})]$$

$$= 4xe^{-x^2} [-2 - 1 + 2x^2] = 4xe^{-x^2} [2x^2 - 3]$$

For maxima & minima

$$\frac{dA}{dx} = 0$$

$$\Rightarrow 2e^{-x^2} [1 - 2x^2] = 0 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

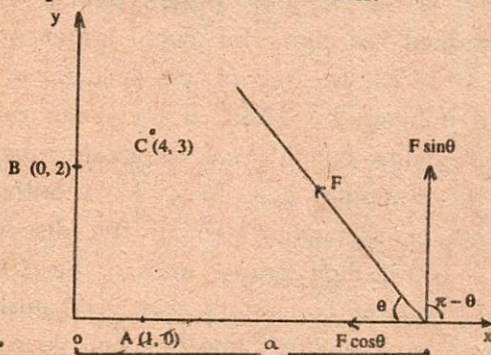
$$\begin{aligned}\text{Now, } \left[\frac{d^2 A}{dx^2} \right]_{x=\frac{1}{\sqrt{2}}} &= 4 \cdot \frac{1}{\sqrt{2}} e^{-\frac{1}{2}[2 \cdot \frac{1}{2} - 3]} \\ &= -\frac{2\sqrt{2}}{\sqrt{e}} \times 2 \\ &\Rightarrow -\frac{4\sqrt{2}}{\sqrt{e}} < 0\end{aligned}$$

Hence, A is maximum when $x = \frac{1}{\sqrt{2}}$

Now, $x = \frac{1}{\sqrt{2}}$ given, $y = e^{-\frac{1}{2}}$

Hence largest area of rectangle $= 2 \cdot \frac{1}{\sqrt{2}} \cdot e^{-\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{e}}$

19. The algebraic sums of the moments of a system of coplanar forces about three points (1, 0), (0, 2) and (2, 3) are 4, 1 and 2 respectively. Determine the tangent of the angle which the resultant force makes with the positive direction of the x-axis.



Soln.

Let F is a force which makes $(\pi - \theta)$ angle with the positive direction of the x-axis

Moment about A $\equiv (1, 0) = 4$

$$\Rightarrow -F \sin \theta (a - 1) = 4 \quad \dots (i)$$

Moment about B $\equiv (0, 2) = 1$

$$\Rightarrow F \cos \theta \cdot 2 - F \sin \theta \cdot a = 1 \quad \dots (ii)$$

Moment about C $\equiv (2, 3) = 2$

$$F \cos \theta \cdot 3 - F \sin \theta (a - 2) = 2 \quad \dots (iii)$$

Subtracting (ii) from (iii)

$$F \cos \theta + 2F \sin \theta = 1 \Rightarrow F = \frac{1}{(\cos \theta + 2 \sin \theta)} \quad \dots (iv)$$

Divide (ii) by (i) we get,

$$\begin{aligned}\frac{F \cos \theta \cdot 2}{F \sin \theta (a - 1)} + \frac{F \sin \theta a}{F \sin \theta (a - 1)} &= \frac{1}{4} \\ \Rightarrow \frac{2}{a - 1} \cot \theta &= \frac{a}{a - 1} - \frac{1}{4} = \frac{4a - a + 1}{4(a - 1)} = \frac{3a + 1}{4(a - 1)}\end{aligned}$$

$$\Rightarrow \cot \theta = \frac{3a + 1}{8} \Rightarrow a = \frac{8 \cot \theta - 1}{3}$$

From eqn. (i), $-F(a - 1) \sin \theta = 4$

$$\Rightarrow +F \left(1 - \frac{8 \cot \theta - 1}{3} \right) \sin \theta = 4$$

$$\Rightarrow F \left(\frac{3 - 8 \cot \theta + 1}{3} \right) \sin \theta = 4$$

$$\Rightarrow F \cdot 4 \cdot \frac{(1 - 2 \cot \theta)}{3} \sin \theta = 4$$

$$\Rightarrow F = \frac{3}{\sin \theta - 2 \cos \theta} \quad \dots (v)$$

From eqns. (iv) and (v)

$$\frac{1}{2 \sin \theta + 2 \cos \theta} = \frac{3}{\sin \theta - 2 \cos \theta}$$

$$\Rightarrow \sin \theta - 2 \cos \theta = 6 \sin \theta + 3 \cos \theta$$

$$\Rightarrow 5 \sin \theta + 5 \cos \theta = 0$$

$$\Rightarrow \sin \theta + \cos \theta = 0$$

$$\Rightarrow \sin \frac{\pi}{4} \cdot \sin \theta + \cos \frac{\pi}{4} \cdot \cos \theta = 0$$

$$\Rightarrow \cos \left(\theta - \frac{\pi}{4} \right) = 0 = \cos \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

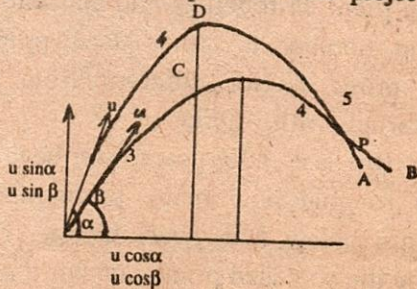
$$\text{Angle} = \pi - \theta = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$$

Tangent of the angle which resultant force makes with the positive direction of the x-axis

$$\tan \frac{\pi}{4} = 1$$

20. Two particles A and B are projected from the same point in the same vertical plane with equal velocities. The particles A and B take 5 and 4 minutes respectively, to common point of their paths. If the particles A and B take 4 & 3 minutes respectively to each their highest points, determine the velocity with which the particles were projected.

Soln.



Let two particles A & B are projected from a point O

with velocity u . Those highest point are D & C respectively and they meet at a point P. Let T_0 is time to reach the highest point.

$$\therefore T_0 = \frac{u \sin \alpha}{g} \Rightarrow 4 = \frac{u \sin \alpha}{g}$$

$$\text{and } 3 = \frac{u \sin \beta}{g}$$

$$\therefore \frac{\sin \alpha}{\sin \beta} = \frac{4}{3} \Rightarrow 3 \sin \alpha = 4 \sin \beta \quad \dots (i)$$

For particle A, $OP = 5 u \cos \alpha$

For particle B, $OP = 4 u \cos \beta$

$$5 u \cos \alpha = 4 u \cos \beta$$

$$\Rightarrow 5 \cos \alpha = 4 \cos \beta \quad \dots (ii)$$

Square & adding equations (i) and (ii) we get,
 $9 \sin^2 \alpha + 25 \cos^2 \alpha = 16 \sin^2 \beta + 16 \cos^2 \beta$

$$\Rightarrow 9 \sin^2 \alpha + 25 - 25 \sin^2 \alpha = 16$$

$$\Rightarrow 16 \sin^2 \alpha = 9 \Rightarrow \sin^2 \alpha = \frac{9}{16}$$

$$\therefore \sin \alpha = \frac{3}{4}$$

$$\therefore 4 = \frac{u \sin \alpha}{g}$$

$$\Rightarrow u = \frac{4g}{\sin \alpha} = \frac{4 \times 9.8 \times 4}{3} = 46.27 \text{ m/s.}$$

So the velocity with which the particles were projected is 46.27 m/s.

LARGEST-KNOWN PRIME NUMBER

The number ($2^{859,433} - 1$) listed in the Guinness Book of Records as the largest-known prime number, has been knocked into numerical obscurity by ($2^{1,257,787} - 1$), a monster of 378,632 digits which has been proven prime by a computer at Cray Research, Wisconsin. It would take about 12 pages of this newspaper to print out the number in full. The old record-holder would have run out after about nine pages.

Prime number Those that can be divided without remainder by no whole numbers other than one and themselves have fascinated mathematicians for more than 2,000 years. Euclid provided the first simple proof that there is an infinity of primes. (If not, must multiply them all together and add one. The resulting number is either itself prime or has a prime divisor different from those you started with, QED).

For the last 100 years, we have even known roughly how many prime numbers there are below any given figure. (This is given by the so-called prime number theorem,

first proved in 1896). Yet despite knowing there is no largest prime, people have continued searching for every larger ones.

In 1772, the record was held by a 10-digit number, by 1884, it had been raised to 20 digits, but the real acceleration began in the computer age. In 1971, months of computer calculation led to the discovery of a 6,002-digit prime, and in the 1980s and 90s Cray computers have been pushing the record higher almost every year.

A spokesman for Cray described prime testing as a torture test for supercomputers. Others might call it a waste of time. Recently, however, the task of factorising large numbers has had important application in computer security.

We had intended to print the new top prime in full, but perhaps for reasons of security the people at Cray have not divulged all its digits. Of course, you can work it out yourself: just take 1, 257, 787 twos, multiply them together and subtract one from the answer. ■

MODEL TEST PAPER

FOR IIT-JEE, ROORKEE, MNR ENTRANCE EXAMS

— Saurav Sharma
Burdwan, West Bengal

PART-A

1. If $\sin \theta = \frac{a^2 + b^2 + c^2}{ab + bc + ca}$

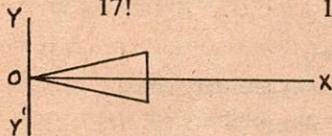
solution for θ

- (a) $a^2 + b^2 + c^2 < 1$
(b) $a = b = c$
(c) $a^2 + b^2 + c^2$ lie between -1 to 1
(d) $bc + ca + ab = 0$

2. $\frac{1 \cdot 2}{2!} + \frac{2 \cdot 2^2}{4!} + \frac{3 \cdot 2^3}{5!} + \dots + \frac{15 \cdot 2^{15}}{17!} =$

- (a) $2 - \frac{16 \cdot 2^{17}}{17!}$ (b) $\frac{2 - 2^{17}}{17!}$
(c) $1 - \frac{16 \cdot 2^{17}}{17!}$ (d) $1 - \frac{2^{16}}{17!}$

3.



is the graph of

- (a) $y \leq x$ (b) $|y| \leq |x|$
(c) $y \leq |x|$ (d) $|y| \leq x$

4. Curve in the complex plane

$\operatorname{Re} \left(\frac{1}{z} \right) = \frac{1}{4}$ is a

- (a) circle of radius 2 (b) circle of radius 1
(c) straight line (d) none

5. If $|x^2 - 7x + 12| > x^2 - 7x + 12$ then

- (a) $x \leq 3$ or $x \geq 4$ (b) $3 \leq x < 4$
(c) $3 < x < 4$ (d) none of 4 or 3

6. Nos. of order of pair (x, y) satisfying $x^2 + 6x + y^2 = 4$ is

- (a) 2 (b) 6
(c) 8 (d) 4

7. If x is real, $y = \frac{1}{2}(e^x - e^{-x})$ then x equals to

- (a) $\log(y + \sqrt{y^2 + 1}) \log(y - \sqrt{y^2 + 1})$
(b) only $\log(y + \sqrt{y^2 + 1})$
(c) $\log(y + \sqrt{y^2 - 1}) \log(y - \sqrt{y^2 - 1})$
(d) only $\log(y + \sqrt{y^2 - 1})$

9. $y = 3 \frac{\sin ax}{\cos bx}$. Find $\frac{dy}{dx}$

10. Find the value of $\int_e^{e^4} \sqrt{\log x} \, dx$ in terms of a

When $\int e^{x^2} dx = a$

11. The smallest integer greater than the real no.

$(\sqrt{5} + \sqrt{3})^{2n}$ is

- (a) $8n$ (b) 4^{2n}
(c) $(\sqrt{5} + \sqrt{3})^{2n} + (\sqrt{5} - \sqrt{3})^{2n}$

12. In a triangle the internal bisector of angle A meets BC at D . $AB = 4$, $AC = 3$, $A = 60^\circ$. Find AD

13. Consider continuous function 'f' in the interval $[0, 1]$ which satisfy

(i) $f(x) \leq \sqrt{5}$ for all $x \in [0, 1]$

(ii) $f(x) \leq \frac{2}{x}$ for all $x \in \left[\frac{1}{2}, 1\right]$

The smallest real no. ' α ' such that inequality $\int f(x) dx \leq \alpha$ holds for any f is

- (a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2} + 2 \log 2$
(c) $2 + 3 \log \frac{\sqrt{5}}{2}$ (d) $2 + \log \frac{\sqrt{5}}{2}$

14. 1 and P be any odd prime nos. Then the nos. of +ve integer K with $1 < K < P$ satisfy; for which K^2 leaves a remainder of 1 when divided by P is

- (a) 1 (b) 2
(c) $P - 1$ (d) $\frac{P - 1}{2}$

15. If $f(x)$ is a polynomial of x and a, b are distinct

real nos. then the remainder in the division of $f(x)$ by $(x-a)(x-b)$ is

- (i) $\frac{(x-a)f(a) - (x-b)f(b)}{a-b}$
 (ii) $\frac{(x-a)f(b) - (x-b)f(a)}{b-a}$

16. Consider the sequence $\alpha_1 = 101$ $\alpha_2 = 10101$ $\alpha_3 = 1010101$ and so on. Then α_k is a composite no. When

- (a) If $K \geq 2$ and 11 divides $10^{K+1} + 1$
 (b) If $K \geq 2$ and 11 divides $10^{K+1} - 1$
 (c) If $K \geq 2$ and $K-2$ is divisible by 3

17. If x, y, z are arbitrary constants satisfying the equation

$$4xy + 6yz + 8zx = 9$$

then the maximum value of xyz is

- (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{4}$
 (c) $\frac{3}{8}$ (d) none

18. Let f and g be two functions defined for all I such that $f(x) \geq 0$ and $g(x) \leq 0$ then

- (a) f is decreasing on I , g is increasing on I
 (b) fg is strictly increasing
 (c) fg is strictly decreasing
 (d) nothing can be said

19. The curve $\frac{2x}{1+x^2}$ has

- (a) Exactly 3 points of inflection separated by a point of maximum and minimum
 (b) Exactly 2 points of inflection and maximum lying between them
 (c) Exactly 2 points of inflection and minimum lying between them

20. Four subjects are there in which marks can be scored from 0 to 10 then the number of ways that total marks obtained will be 21 is

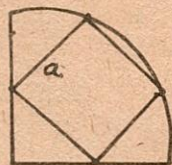
- (a) 220 (b) 760
 (c) 450 (d) 1360

21. Find the number of ways that product of any number of consecutive numbers from 1 to 8 which can be divisible by 5 is

- (a) $2 \cdot 7!$ (b) $8 \cdot 7!$
 (c) $7!$ (d) $4 \cdot {}^7C_4 \cdot 5! \cdot 3! \cdot 4!$

22. In a quarter circle a square is inscribed in such a way that its vertices lie on the figure as shown. Sides of square are 'a'

Find the radius of the circle.



23. If the bisectors of a quadrilateral form a quadrilateral then the sum of the cosine of all the angles of new quadrilateral is

- (a) dependent of previous quadrilateral
 (b) non dependent of previous quadrilateral
 (c) A constant
 (d) equal to the sum of sine of all the previous angles of the quadrilateral.

24. $f(x) = a^2 + 2x$ when $x \leq 0$
 $= ax + b$ when $x > 0$

f is differentiable when at $x = 0$ when

- (a) $a = 2$ $b = 0$
 (b) $a = 0$ $b = 0$
 (c) $a = 2$ $b = \text{any arbitrary const.}$
 (d) $a = 0$ $b = \text{any arbitrary const.}$

PART-B

1. Find the area of the region in any plane bounded by graphs of $y = x^2$, $x + y = 2$ and $y = -\sqrt{x}$

2. Show that if x is any odd integer then $x^5 - x$ is divisible by 80.

3. Using calculus sketch the graph of the following function on a plane paper

$$f(x) = \frac{5 - 3x^2}{1 - x^2}$$

4. $P(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0$ be a polynomial with integer co-efficients such that $P(0) \times P(1)$ are odd integers. Show that

- (a) $P(x)$ does not have any even integer root
 (b) $P(x)$ does not have odd integer root

5. Let ABC be any triangle right angled at A , with D any point on the side AB . The line DE is drawn

parallel to BC to meet AC at point E. F is the foot of the perpendicular drawn from E to BC. If $AD = x_1$, $DB = x_2$, $BF = x_3$, $EF = x_4$ and $AE = x_5$, then show that

$$\frac{x_1}{x_5} + \frac{x_2}{x_5} = \frac{x_1 x_3 + x_4 x_5}{x_3 x_5 - x_1 x_4}$$

6. If regular five pointed star is inscribed in a circle of radius 'r'. Show that the area of the star is

$$\frac{10r^2 - \tan\left(\frac{\pi}{10}\right)}{3 - \tan^2\left(\frac{\pi}{10}\right)}$$



7. (a) In the identity

$$\frac{n!}{x(x+1)(x+2)\dots(x+n)} = \sum_{k=0}^n \frac{A_k}{x+k}$$

Prove that $A_k = (-1)^k \binom{n}{k}$

(b) Deduce that

$$\binom{n}{0} \frac{1}{1 \cdot 2} - \binom{n}{1} \frac{1}{2 \cdot 3} + \binom{n}{2} \frac{1}{3 \cdot 4} + \dots + \dots + (-1)^n \binom{n}{n} \frac{1}{n+1} = \frac{1}{n+2}$$

8. (a) Given 'm' identical symbols say Hs. Show that the number of ways in which you can distribute them in k box is marked 1, 2, ..., k, so that no box goes empty is $\binom{m-1}{k-1}$

(b) In an arrangement of m - H's and n - T's an uninterrupted sequence of one kind of symbol is called a run.

equating the arrangement HHH T HH TTT H of 6H and 4T opens with an H run of length 3, followed by a T run of length 1, an H run of length 2 a T run of length 3 and finally an H run of length 1.

Find the number of arrangements of mH and n T in which there are exactly K H runs.

9. A pair of complex number Z_1, Z_2 is used to have properly P if for every complex number Z we can find real numbers 'r' and 's' such that $Z = rZ_1 + sZ_2$. Show that a pair of (Z_1, Z_2) have properly P if and only if the points (Z_1, Z_2) and O on the complex plane are not collinear.

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Business schools command respect and reverence in the mindsets of the intelligentsia of any country. The aura and mystique surrounding the MBA degree is a cleverly cultivated myth which has grown by leaps and bounds. The rapid mushrooming of business schools, much like the teeming of mosquitoes after the first rain, has in some way jolted this myth. Suddenly, these schools are omnipresent. Recognised, unrecognised; backed by a university or autonomous, yearly, semester, trimester, and permutations and combinations of all these. Though the phenomenon of universalisation of business schools has made them more visible, the aura enveloping them continues unabated.

Many things happen simultaneously, as soon as you enter any business school. The reaction of society, particularly your peer group, does a *volte face*. Ambiguity regarding your future turns into certainty (the increased rate of marriage proposals bear ample testimony to this!), ditherence about your capability turns into healthy respect, and suspicion of foolhardiness gives way to intellectualism. And, believe it or not all this is a positive ego boost for the centre of attraction-the entrant.

His own perception regarding himself changes. He feels more confident and assured. A major contributor to this new-found confidence is the certainty of a good job. The demand for MBAs, despite burgeoning business schools, far outstrips their supply. Organisations' preference for MBAs in their hierarchy, at all possible levels, assures the young student of a niche somewhere.

Another thing happens. He suddenly becomes more aware of his surroundings. He might be unaware of the bustling corporate world, or the fluctuations of the stock market earlier, but the new state-of-affairs make way for a heightened sense of awareness of one's environment. His curiosity regarding his future workplace manifests itself through in-depth persual of newspapers, periodicals,

magazines, and textbooks. He is suddenly exposed to a wealth of data. And, in his anxiety, he wants to digest it all. In addition, at a deeper level, there is the fact of personality development at a subtle pace in a slow and steady manner. His exposure to information sources, interaction with teachers and peer group, and direct contact with the corporate world via summer training - all contribute to the blossoming of his personality.

A new phase of life confronts him at the end of two years. Gone is the protected, serene, and stable environment of the business school. The days ahead give him an inkling of the harsh rivalry, cut-throat competition, and heightened expectations. With the tag of MBA on him, he is expected to deliver each time, time after time. A unique development precedes this - that of displacement. Hitherto, even after staying away from home because of education, a sense of a home beckoning you always remains. Come vacations and one longs to be in one's home where a sense of protection and care engulfs you. But the fresher MBA is expected to perform a permanent eloping. Gone is the notion of a home and the sense of anxiety associated with homecoming. To survive in the corporate world requires total dedication in addition to skills. The concept of locational constraints somehow loses its meaning. It's like a make-or-break chance. No body is willing to hamper his chances of success by these petty barriers. It's now or never. And so each one of them joins the bandwagon - forgetting everything about dwelling, food, or society. The concentration is on a career which overshadows everything else.

And once the rat race starts, there is no looking back. It is a mad rush to the top echelons of the organisation. Since the space at the top is usually narrow, only the fittest survive - those who have excelled in gaining recognition, creating favourable impact on their bosses, and backing all this with solid factual results. In the process, the first thing to be sacrificed is family life. Other sacrificial areas

include peace, happiness and time. They are respectively replaced by tensions and worries, disillusionment, and an extremely busy life. By the time the MBAs stop to ponder about all this, it is, perhaps, too late for them. The system has caught up with them and they are trapped in the vortex of the overall environment enveloping them.

Will this rat race for name, fame, recognition, and money ever end? Will MBAs, fetish for status and recognition continue to drive them towards the materialistic world? How long will money continue to rule over values and basic human endeavours? How long will the dichotomy between organisation's family and the personal family continue to get settled in favour of the former? Career is everybody's passion. But is there a feeble voice which places internal happiness, soulful satisfaction, and personality enhancement at par with career advancement? Certainly, these questions are not for the fresher MBA. He is bound by the tradition and customs of MBAs all round him.

Isn't it ironical that such and related questions strike him only later in his career. A sense of despondency and the realisation of "could-have-done-something-different" bears ample proof to the importance of addressing all these questions at the start and not at the middle of your career.

Change is the essence of life. A conscious shift in the attitude of fresh MBAs towards their careers can go a long way in adding happiness to their own lives and confirming their status as better human beings in society.

Contd. from page No.. 58

$$= 2 \left[\left\{ \frac{(x-1) \sqrt{1-(x-1)^2}}{2} + \frac{1}{2} \sin^{-1}(x-1) \right\} \right]_0^{1/2} \\ + \left[\frac{x \sqrt{1-x^2}}{2} + \frac{1}{2} \sin^{-1} x \right]_0^{1/2} \\ = \left[\frac{-\sqrt{3}}{4} + \frac{\pi}{3} + \frac{\pi}{3} - \frac{\sqrt{3}}{4} \right] = 2 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right)$$

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HIGHER ALGEBRA PROBLEMS

— V. Vasudevan Potti, Trivandrum

1. If $\frac{3x+2y}{3a-2b} = \frac{3y+2z}{3b-2c} = \frac{3z+2x}{3c-2a}$ then will

$$5(x+y+z)(5c+4b-3a) = (9x+8y+13z)(a+b+c).$$

Soln.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ then each ratio equals $\frac{a+c+e}{b+d+f}$

$$\frac{a+c+e}{b+d+f} = \frac{kb+kd+kf}{b+d+f} = \frac{k(b+d+f)}{b+d+f} = k$$

Similarly each ratio equals $\frac{a+2c+3c}{b+2d+3f}$

Applying this result each fraction equals

$$= \frac{3x+2y+3y+2z+3z+2x}{3a-2b+3b-2c+3c-2a} = \frac{5(x+y+z)}{a+b+c}$$

Again each fraction equals

$$= \frac{3x+2y+2(3y+2z)+3(3z+2x)}{3a-2b+2(3b-2c)+3(3c-2a)} \\ = \frac{3x+2y+6y+4z+9z+6x}{3a-2b+6b-4c+9c-6a} = \frac{9x+8y+13z}{5c+4b-3a}$$

Equating the two ratios

$$\frac{5(x+y+z)}{a+b+c} = \frac{9x+8y+13z}{5c+4b-3a}$$

$$\therefore 5(x+y+z)(5c+4b-3a) = (9x+8y+13z)(a+b+c).$$

2. With 17 consonants and 5 vowels how many words of four letters can be formed having 2 different vowels in the middle and 1 consonant repeated or different at each end.

Soln.

First place is filled using any of 17 consonants, second place using any of 5 vowels, third place using remaining 4 vowels and last place using 17 consonants (since repetition of consonants is allowed)

$$\therefore \text{number of 4 letter words} = 17 \times 5 \times 4 \times 17 \\ = 289 \times 20 = 5780.$$

3. A question was lost on which 600 persons had

voted. The same persons having voted again on the same question it was carried by twice as many as it was before lost by and the new majority vote was to the former as 8 to 7. How many changed their mind

Soln.

Let x be the number of persons voted in favour of the question in the first time

$$\therefore \text{No. of persons voted against} = 600 - x$$

$$\text{Majority by which the question was lost} = (600 - x) - x \\ = 600 - 2x$$

Let y persons changed their mind for the second time

Then the number of persons voted in favour of the question $= x + y$

$$\text{No. of persons voted against} = 600 - (x + y)$$

$$\text{Majority by which the question was carried by} \\ = (x + y) - (600 - (x + y)) \\ = 2(x + y) - 600$$

Given,

$$\text{Majority by which the question was carried by} \\ = 2 \times \text{majority by which it was lost}$$

$$2(x + y) - 600 = 2(600 - 2x)$$

$$2x + 2y - 600 = 1200 - 4x$$

$$6x + 2y = 1800 \quad 3x + y = 900$$

$$\text{Ratio of new majority vote to the old} = 8 : 7$$

$$\frac{x+y}{600-x} = \frac{8}{7} \Rightarrow 7x + 7y = 4800 - 8x$$

$$\text{and } 15x + 7y = 4800 \quad \dots (i)$$

$$15x + 7y = 4800$$

$$15x + 5y = 4500 \quad \dots (ii)$$

$$\text{Subtracting, } \frac{2y}{2y} = \frac{300}{300} \Rightarrow y = 150$$

$\therefore$ Number of persons who changed their mind $= 150$.

4. Show that

$$\log \frac{(1+x)^{1-x/2}}{(1-x)^{1+x/2}} = x + \frac{5x^3}{2 \cdot 3} + \frac{9x^5}{4 \cdot 5} + \frac{13x^7}{6 \cdot 7} + \dots$$

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Soln.

$$\begin{aligned} \text{LHS} &= \log \frac{(1+x)^{1-x/2}}{(1-x)^{1+x/2}} \\ &= \log (1+x)^{1-x/2} - \log (1-x)^{1+x/2} \\ &= \frac{1-x}{2} \log (1+x) - \frac{1+x}{2} \log (1-x) \\ &= \frac{1}{2} [\log (1+x) - \log (1-x)] - \end{aligned}$$

$$\begin{aligned} &\frac{x}{2} [\log (1+x) + \log (1-x)] \\ \Rightarrow &\frac{1}{2} \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \right] \\ &- \frac{x}{2} \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots - x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \right] \\ &= \frac{1}{2} \left[2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \dots \right] + \\ &\quad \frac{x}{2} \left[\frac{2x^2}{2} + \frac{2x^4}{4} + \frac{2x^6}{6} + \dots \right] \\ &= x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^3}{2} + \frac{x^5}{4} + \frac{x^7}{6} + \dots \\ &= x + \left(\frac{1}{2} + \frac{1}{3} \right) x^3 + \left(\frac{1}{4} + \frac{1}{5} \right) x^5 + \dots \\ &= x + \frac{5}{2 \cdot 3} x^3 + \frac{9}{4 \cdot 5} x^5 + \frac{13}{6 \cdot 7} x^7 + \dots \end{aligned}$$

5. Solve the equations

$$\begin{aligned} \text{(i)} \quad &\sqrt[4]{(a+x)^2 + 2} \sqrt[4]{(a-x)^2} = 3 \sqrt[4]{a^2 - x^2} \\ \text{(ii)} \quad &(x-a)^{1/2} (x-b)^{1/2} - (x-c)^{1/2} (x-d)^{1/2} = \\ &\quad (a-c)^{1/2} (b-d)^{1/2} \end{aligned}$$

Soln.

(i) Given,

$$\sqrt[4]{(a+x)^2 + 2} \sqrt[4]{(a-x)^2} = 3 \sqrt[4]{a^2 - x^2}$$

Dividing, by $\sqrt[4]{a^2 - x^2}$

$$\sqrt[4]{\frac{a+x}{a-x}} + 2 \sqrt[4]{\frac{a-x}{a+x}} = 3$$

$$\text{Put, } y = \sqrt[4]{\frac{a+x}{a-x}}$$

$$\text{then } y + \frac{2}{y} = 3, \quad y^2 + 2 = 3y \quad y^2 - 3y + 2 = 0$$

$$(y-1)(y-2) = 0 \Rightarrow y = 1 \text{ or } 2$$

$$\text{i.e. } \sqrt[4]{\frac{a+x}{a-x}} = 1 \quad \text{or} \quad \sqrt[4]{\frac{a+x}{a-x}} = 2$$

$$\frac{a+x}{a-x} = 1$$

$$\text{or } \frac{a+x}{a-x} = 2^m$$

$$a+x = a-x$$

$$2x = 0$$

$$x = 0$$

$$a+x = a \cdot 2^m - x \cdot 2^m$$

$$(2^m + 1)x = a \cdot 2^m - a$$

$$x = \frac{a(2^m - 1)}{(2^m + 1)}$$

$$\begin{aligned} \text{(ii)} \quad &(x-a)^{1/2} (x-b)^{1/2} - (x-c)^{1/2} (x-d)^{1/2} = \\ &\quad (a-c)^{1/2} (b-d)^{1/2} \\ &= [(x-c) - (x-a)]^{1/2} [(x-d) - (x-b)]^{1/2} \end{aligned}$$

Squaring both sides,

$$\begin{aligned} &(x-a)(x-b) + (x-c)(x-d) \\ &\quad - 2\sqrt{(x-a)(x-b)(x-c)(x-d)} \\ &= [(x-c) - (x-a)][(x-d) - (x-b)] \\ &= (x-c)(x-d) - (x-b)(x-c) - (x-a)(x-d) \\ &\quad + (x-a)(x-b) \\ \therefore &(x-c)(x-b) + (x-a)(x-d) = \\ &\quad 2\sqrt{(x-a)(x-b)(x-c)(x-d)} \end{aligned}$$

Squaring,

$$\begin{aligned} &(x-c)^2(x-b)^2 + (x-a)^2(x-d)^2 + \\ &\quad 2(x-a)(x-b)(x-c)(x-d) \\ &= 4(x-a)(x-b)(x-c)(x-d) \\ &(x-c)^2(x-b)^2 + (x-a)^2(x-d)^2 - \\ &\quad 2(x-a)(x-b)(x-c)(x-d) = 0 \\ \therefore &[(x-c)(x-b) - (x-a)(x-d)]^2 = 0 \\ \text{i.e. } &(x-c)(x-b) = (x-a)(x-d) \\ &x^2 - bx - cx + bc = x^2 - ax - dx + ad \\ &x(a+d-b-c) = ad-bc \\ &x = \frac{ad-bc}{a-b-c+d} \end{aligned}$$

6. Prove that

$$\sqrt[3]{4} = 1 + \frac{2}{6} + \frac{2 \cdot 5}{6 \cdot 12} + \frac{2 \cdot 5 \cdot 8}{6 \cdot 12 \cdot 18} + \dots$$

Soln.

$$\text{RHS} = 1 + \frac{2}{1} \left(\frac{1}{6} \right) + \frac{2 \cdot 5}{1 \cdot 2} \left(\frac{1}{6} \right)^2 + \frac{2 \cdot 5 \cdot 8}{1 \cdot 2 \cdot 3} \left(\frac{1}{6} \right)^3 + \dots$$

$$p = 2, \quad q = 5 - 2 = 3 \quad \text{and} \quad \frac{x}{q} = \frac{1}{6} \Rightarrow x = \frac{1}{2}$$

$$\text{RHS} = (1-x)^{-p/q} = \left(1 - \frac{1}{2} \right)^{-2/3}$$

$$\Rightarrow \left(\frac{1}{2} \right)^{-2/3} = 2^{2/3} = \sqrt[3]{2^2} = \sqrt[3]{4} = \text{LHS.}$$

$$\text{7. Solve } \sqrt[3]{6(5x+6)} - \sqrt[3]{5(6x-11)} = 1$$

Soln.

$$\sqrt[3]{6(5x+6)} - \sqrt[3]{5(6x-11)} = 1$$

Cubing,

$$\begin{aligned}
 & \frac{6(5x+6) - 5(6x-11)}{3 \sqrt[3]{6(5x+6) 5(6x-11)}} \times (\sqrt[3]{6(5x+6)} - \sqrt[3]{5(6x-11)}) \\
 & 30x + 36 - 30x + 55 - 3 \sqrt[3]{6(5x+6) 5(6x-11)} \times 1 = 1 \\
 & 91 - 3 \sqrt[3]{30(30x^2 + 36x - 55x - 66)} = 1 \\
 & 90 = 3 \sqrt[3]{30(30x^2 - 19x - 66)} \\
 & \text{Cubing, } 30 = 30(30x^2 - 19x - 66) \\
 & 900 = 30x^2 - 19x - 66 \\
 & \text{i.e. } 30x^2 - 19x - 966 = 0 \\
 & 30x^2 + 161x - 180x - 966 = 0 \\
 & x(30x + 161) - 6(30x + 161) = 0 \\
 & (30x + 161)(x - 6) = 0 \\
 & x = 6 \text{ or } x = -\frac{161}{30}
 \end{aligned}$$

8. The arithmetic mean between m and n and the geometric mean between a and b are each equal to

$$\frac{ma + nb}{m + n}, \text{ find } m \text{ and } n \text{ in terms of } a \text{ and } b.$$

Soln.

$$\text{Given, } \frac{m+n}{2} = \sqrt{ab} = \frac{ma+nb}{m+n}$$

$$m+n = 2\sqrt{ab} \quad \dots (i) \text{ by equating first two}$$

By equating last two

$$\frac{ma+nb}{m+n} = \sqrt{ab}$$

$$ma+nb = (m+n) \cdot \sqrt{ab} = 2\sqrt{ab} \cdot \sqrt{ab} = 2ab \text{ by (i)}$$

$$m+n = 2\sqrt{ab} \quad \dots (i)$$

$$ma+nb = 2ab \quad \dots (ii)$$

(i) $\times a$ gives

$$ma + na = 2a\sqrt{ab}$$

$$ma + nb = 2ab$$

$$n(a-b) = 2a\sqrt{b}(\sqrt{a}-\sqrt{b})$$

$$n = \frac{2a\sqrt{b}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

(ii) - (i) $\times b$

$$ma + nb = 2ab$$

$$mb + nb = 2b\sqrt{ab}$$

Subtracting,

$$\begin{aligned}
 m(a-b) &= 2b\sqrt{a}(\sqrt{a}-\sqrt{b}) \\
 m &= \frac{2\sqrt{ab}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} \\
 &= \frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}
 \end{aligned}$$

9. If x, y, z are such that their sum is constant and

if $(z+x-2y)(x+y-2z)$ varies as yz prove that $2(y+z) - x$ varies as yz

Soln.

Let $x + y + z = c$ (a constant)

Given, $(z+x-2y)(x+y-2z) = myz$

$$(c-y-2y)(c-z-2z) = myz$$

$$(c-3y)(c-3z) = myz$$

$$c^2 - 3c(y+z) + 9yz = myz$$

$$(9-m)yz = c[3(y+z) - c]$$

$$= c[3(y+z) - (x+y+z)]$$

$$= c[2y+2z-x]$$

$$\therefore \frac{2y+2z-x}{yz} = \frac{9-m}{c} \text{ (a constant)}$$

$$\therefore (2y+2z-x) \propto (yz).$$

10. Prove that if n is greater than 3

$$1 \cdot 2 nC_r - 2 \cdot 3 nC_{r-1} + 3 \cdot 4 nC_{r-2} - \dots$$

$$+ (-1)^r (r+1)(r+2) = 2 \cdot (n-3)C_r$$

Soln.

Consider,

$$(1+x)^n = 1 + nC_1x + nC_2x^2 + nC_3x^3 + \dots + nC_rx^r + \dots$$

$$(1+x)^{-3} = 1 - 3x + \frac{3 \cdot 4}{1 \cdot 2} x^2 - \frac{4 \cdot 5}{1 \cdot 2} x^3 + \dots$$

$$+ (-1)^n \frac{(r+1)(r+2)}{1 \cdot 2} + \dots$$

Multiplying the two series,

$$(1+x)^{n-3} =$$

$$(1 + nC_1x + nC_2x^2 + nC_3x^3 + \dots + nC_rx^r + \dots)$$

$$\left(1 - 3x + \frac{3 \cdot 4}{1 \cdot 2} x^2 - \frac{4 \cdot 5}{1 \cdot 2} x^3 + \dots + (-1)^r \frac{(r+1)(r+2)}{1 \cdot 2} x^r + \dots \right)$$

Equating coefficients of x^r

$$\text{LHS} = (n-3)C_r$$

$$\text{RHS} = 1 \cdot nC_r - 3 \cdot nC_{r-1} + \frac{3 \cdot 4}{1 \cdot 2} nC_{r-2} - \dots + (-1)^r \frac{(r+1)(r+2)}{1 \cdot 2}$$

$$=$$

$$1 \cdot 2 nC_r - 2 \cdot 3 nC_{r-1} + 3 \cdot 4 nC_{r-2} - \dots + (-1)^r (r+1)(r+2)$$

$$\therefore 2(n-3)C_r = 1 \cdot 2 nC_r - 2 \cdot 3 nC_{r-1} + 3 \cdot 4 nC_{r-2} - \dots + (-1)^r (r+1)(r+2).$$

for IIT/CBSE

Aspirants !

DEAR SIR, I obtained your correspondence study material through Paradise Institute, Patna. The study material, least to say was really very instructive and helpful. The courses enabled me to secure 14th rank in CBSE(PMT) 1996, AIR 150 in IIT-96, 6th rank in PMT(MP) 1996, 18th rank in PET(MP) 96, 7th rank in P.P.T(MP) 96.

Nishith Kumar Singh, Bhilainagar (Distt. Durg) M.P.

Last year we distributed about 100 sets of our study-material (Cyclostyled notes) through friends and friendly Institutes (including Paradise Institute, Patna). Letters of response (one of which is presented above) speak volumes about the quality of the material.

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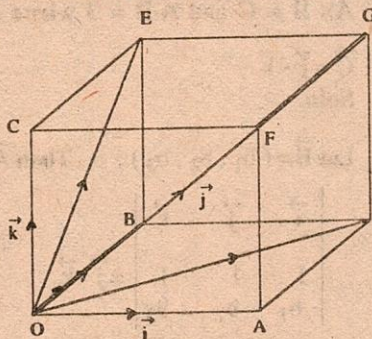
Dr. Md. Motiur Rahman
Darbhanga, Bihar

1. Three vectors of magnitudes a , $2a$, $3a$ meet in a point and their direction are along the diagonals of three adjacent faces of a cube. Determine their resultant R and its direction cosines. Prove also that sum of three vectors determined by the diagonals of three adjacent faces of a cube passing through the same corner, the vectors being directed from the corner, is twice the vector determined by the diagonal of the cube.

Soln.

Let OABCDEFG be a cube whose side is of unit length.

Let $\vec{i}, \vec{j}, \vec{k}$ be unit vectors along OA, OB and OC respectively.



Then, $\vec{OD} = \vec{OA} + \vec{AD} = \vec{OA} + \vec{OB} = \vec{i} + \vec{j}$

Similarly, $\vec{OE} = \vec{j} + \vec{k}$ and $\vec{OF} = \vec{k} + \vec{i}$

$$\therefore |\vec{OD}| = |\vec{OE}| = |\vec{OF}| = (1^2 + 1^2)^{1/2} = \sqrt{2}$$

Therefore, the unit vectors along OD, OE and OF are respectively

$$\frac{\vec{i} + \vec{j}}{\sqrt{2}}, \frac{\vec{j} + \vec{k}}{\sqrt{2}}, \frac{\vec{k} + \vec{i}}{\sqrt{2}}$$

Hence,

$$\begin{aligned} \vec{R} &= a(\vec{i} + \vec{j})/\sqrt{2} + 2a(\vec{j} + \vec{k})/\sqrt{2} + 3a(\vec{k} + \vec{i})/\sqrt{2} \\ &= (4a/\sqrt{2})\vec{i} + (3a/\sqrt{2})\vec{j} + (5a/\sqrt{2})\vec{k} \quad \dots (1) \end{aligned}$$

$$\begin{aligned} \therefore R = |\vec{R}| &= \frac{a}{\sqrt{2}}(4^2 + 3^2 + 5^2)^{1/2} = \frac{a}{\sqrt{2}} \cdot 5\sqrt{2} \\ &= 5a \quad \dots (2) \end{aligned}$$

The direction cosines of R are $4/5\sqrt{2}$, $3/5\sqrt{2}$, $5/5\sqrt{2}$
..... (3)

Further we have,

$$\vec{OG} = \vec{OA} + \vec{AD} + \vec{DG} = \vec{i} + \vec{j} + \vec{k} \quad \dots (4)$$

$$\begin{aligned} \text{And, } \vec{OD} + \vec{OE} + \vec{OF} &= \vec{i} + \vec{j} + \vec{j} + \vec{k} + \vec{k} + \vec{i} \\ &= 2(\vec{i} + \vec{j} + \vec{k}) = 2\vec{OG}. \end{aligned}$$

2. Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ be the position vectors of points $P_1, P_2, P_3, \dots, P_n$ relative to an origin O. Show that if the vector equation

$a_1\vec{r}_1 + a_2\vec{r}_2 + \dots + a_n\vec{r}_n = 0$, then a similar equation will also hold good with respect to any other origin if $a_1 + a_2 + a_3 + \dots + a_n = 0$

Soln.

Let O' be any other origin.

$$\text{Let } \vec{O'O} = \vec{r}_0, \vec{O'P}_1$$

$$= \vec{r}_1,$$

$$\vec{O'P}_2 = \vec{r}_2, \dots, \vec{O'P}_n = \vec{r}_n$$

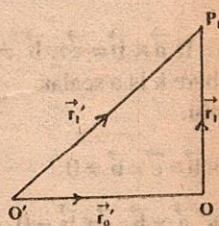
$$\text{Then } \vec{r}_1 = \vec{O'P}_1 = \vec{O'O} + \vec{OP}_1 = \vec{r}_0 + \vec{r}_1$$

Similarly,

$$\vec{r}_2 = \vec{r}_0 + \vec{r}_2$$

$$\vec{r}_n = \vec{r}_0 + \vec{r}_n$$

$$\begin{aligned} \therefore a_1\vec{r}_1 + a_2\vec{r}_2 + \dots + a_n\vec{r}_n &= a_1(\vec{r}_0 + \vec{r}_1) + a_2(\vec{r}_0 + \vec{r}_2) + \dots + a_n(\vec{r}_0 + \vec{r}_n) \\ &= (a_1 + a_2 + \dots + a_n)\vec{r}_0 + a_1\vec{r}_1 + a_2\vec{r}_2 + \dots + a_n\vec{r}_n \end{aligned}$$



$$= (a_1 + a_2 + \dots + a_n) \vec{r}_0;$$

$$\therefore a_1 \vec{r}_1 + a_2 \vec{r}_2 + \dots + a_n \vec{r}_n = 0 \text{ (given)}$$

$$= 0; \text{ provided } a_1 + a_2 + \dots + a_n = 0.$$

3. The position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ of four points A, B, C and D respectively on a plane area such that

$$(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) = (\vec{b} - \vec{d}) \cdot (\vec{c} - \vec{a}) = 0$$

Show that D is orthocentre of the ΔABC .

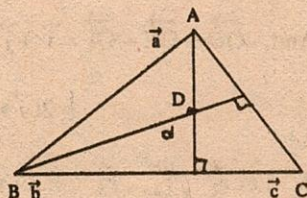
Soln.

We have,

$$\vec{a} - \vec{d} = \text{p.v. of A}$$

$$\text{p.v. of D} = \vec{DA}$$

$$\text{etc.}$$



$$\therefore (\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) = (\vec{b} - \vec{d}) \cdot (\vec{c} - \vec{a}) = 0$$

$$\Rightarrow \vec{DA} \cdot \vec{CB} = 0 \text{ and } \vec{DB} \cdot \vec{AC} = 0$$

$$\Rightarrow AD \perp BC \text{ and } BD \perp CA.$$

Hence, D is the orthocentre of the ΔABC .

4. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{b} \neq 0$, then show that $\vec{a} - \vec{c} = k\vec{b}$, where k is a scalar

Soln.

$$\vec{a} \times \vec{b} = \vec{c} \times \vec{b} \neq 0$$

$$\Rightarrow \vec{a} \times \vec{b} - \vec{c} \times \vec{b} = 0 \Rightarrow (\vec{a} - \vec{c}) \times \vec{b} = 0$$

$$\Rightarrow \vec{a} - \vec{c} \text{ and } \vec{b} \text{ are parallel}$$

$$\Rightarrow \vec{a} - \vec{c} = k\vec{b}, \text{ where k is a scalar.}$$

5. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors, prove that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \end{vmatrix} = 0$$

Soln.

Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, hence we have

$$x\vec{a} + y\vec{b} + z\vec{c} = 0 \quad \dots\dots (1)$$

Taking dot product of (1) with $\vec{a}$ and $\vec{b}$, we get,

$$x\vec{a} \cdot \vec{a} + y\vec{a} \cdot \vec{b} + z\vec{a} \cdot \vec{c} = 0 \quad \dots\dots (2)$$

$$x\vec{b} \cdot \vec{a} + y\vec{b} \cdot \vec{b} + z\vec{b} \cdot \vec{c} = 0 \quad \dots\dots (3)$$

Eliminating x, y, z from (1), (2) and (3), we get,

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \end{vmatrix} = 0.$$

6. Find the vector $\vec{B}$ satisfying the equations

$$\vec{A} \times \vec{B} = \vec{C} \text{ and } \vec{A} \cdot \vec{B} = 3 \text{ where } \vec{A} = \vec{i} + \vec{j} + \vec{k} \text{ and}$$

$$\vec{C} = \vec{j} - \vec{k}$$

Soln.

$$\text{Let } \vec{B} = (b_1, b_2, b_3), \text{ Then } \vec{A} \times \vec{B} = \vec{C}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix} = \vec{j} - \vec{k}$$

$$\Rightarrow (b_3 - b_2)\vec{i} + (b_1 - b_3)\vec{j} + (b_2 - b_1)\vec{k} = \vec{j} - \vec{k}$$

Since $\vec{i}, \vec{j}, \vec{k}$ are non-coplanar vectors, hence we have, $b_3 - b_2 = 0$

$$\Rightarrow b_2 = b_3 \quad \dots\dots (1) \text{ and}$$

$$b_1 - b_3 = 1 \Rightarrow b_1 = 1 + b_3 \quad \dots\dots (2)$$

$$\therefore \vec{B} = (1 + b_3, b_3, b_3) \quad \dots\dots (3)$$

$$\text{Now, } \vec{A} \cdot \vec{B} = 3 \Rightarrow 1 + b_3 + b_3 + b_3 = 3$$

$$\Rightarrow b_3 = 2/3 \text{ and } 1 + b_3 = 5/3$$

$$\text{Hence, } \vec{B} = (5/3, 2/3, 2/3).$$

7. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}$ and

$$\vec{C} = 4\vec{i} - 3\vec{j} + 7\vec{k}. \text{ Find vector } \vec{R} \text{ which satisfies}$$

$$\vec{R} \times \vec{B} = \vec{C} \times \vec{B} \text{ and } \vec{R} \cdot \vec{A} = 0.$$

Soln.

$$\text{Let } \vec{R} = x\vec{i} + y\vec{j} + z\vec{k} \quad \text{Then } \vec{R} \cdot \vec{A} = 0$$

$$\Rightarrow (x\vec{i} + y\vec{j} + z\vec{k}) \cdot (2\vec{i} + \vec{k}) = 0$$

$$\Rightarrow 2x + z = 0 \Rightarrow z = -2x \quad \dots\dots\dots(1)$$

$$\text{And } \vec{R} \times \vec{B} = \vec{C} \times \vec{B}$$

$$\Rightarrow (x\vec{i} + y\vec{j} + z\vec{k}) \times (\vec{i} + \vec{j} + \vec{k}) =$$

$$(4\vec{i} - 3\vec{j} + 7\vec{k}) \times (\vec{i} + \vec{j} + \vec{k})$$

$$\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow (y - z)\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}$$

$$= (-3 - 7)\vec{i} + (7 - 4)\vec{j} + (4 + 3)\vec{k} = -10\vec{i} + 3\vec{j} + 7\vec{k}$$

Since $\vec{i}, \vec{j}, \vec{k}$ are non-coplanar vectors, hence we have

$$y - z = -10, z - x = 3 \text{ and } x - y = 7 \quad \dots\dots\dots(2)$$

Solving (1) and (2), we get $x = -1, y = -8$ and $z = 2$.

$$\text{Hence, } \vec{R} = -\vec{i} - 8\vec{j} + 2\vec{k}.$$

8. Let x, y, z be distinct non-negative numbers. If

the vectors $x\vec{i} + x\vec{j} + z\vec{k}, \vec{i} + \vec{k}$ and $z\vec{i} + z\vec{j} + z\vec{k}$ lie in a plane, then prove that z is the GM between x and y .

Soln.

Since $x\vec{i} + y\vec{j} + z\vec{k}, \vec{i} + \vec{k}$ and $z\vec{i} + z\vec{j} + y\vec{k}$ lie in a plane, i.e. are coplanar, hence we have

$$\begin{vmatrix} x & x & z \\ 1 & 0 & 1 \\ z & z & y \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & x & z \\ 1 & 0 & 1 \\ 0 & z & y \end{vmatrix} = 0, c_1 - c_2$$

$$\Rightarrow - \begin{vmatrix} x & z \\ z & y \end{vmatrix} = 0$$

$$\Rightarrow z^2 = xy \Rightarrow z \text{ is the GM between } x \text{ and } y.$$

9. In a triangle OAB, E is the mid-point of OB and D is a point on AB such that AD:DB = 2:1. If OD

and AE intersect at P, determine the ratio OP:PD, using vector method.

Soln.

$$\text{Let } \vec{OA} = \vec{a}, \vec{OB} = \vec{b}$$

$$\text{so that } \vec{OE} = \vec{b}/2$$

$$\text{and } \vec{OD} = (\vec{a} + 2\vec{b})/3$$

$$\text{The equation of OP is } \vec{r} = u(\vec{a} + 2\vec{b})/3 \quad \dots\dots\dots(1)$$

The equation of AP is

$$\vec{r} = \vec{OA} + v(\vec{b}/2 - \vec{a}) = (1 - v)\vec{a} + \frac{v}{2}\vec{b} \quad \dots\dots\dots(2)$$

Hence, for P, we have

$$u(\vec{a} + 2\vec{b})/3 = (1 - v)\vec{a} + \frac{v}{2}\vec{b} \quad \dots\dots\dots(3)$$

Since, $\vec{a}$ and $\vec{b}$ are non-collinear vectors, hence we have, $u/3 = 1 - v$ and $2u/3 = v/2$

Solving, we get $u = 3/5$ and $v = 4/5$

$$\therefore \vec{OP} = \frac{3}{5} \cdot \frac{\vec{a} + 2\vec{b}}{3} = \frac{\vec{a} + 2\vec{b}}{5}$$

$$\therefore \frac{OP}{OD} = \frac{|\vec{OP}|}{|\vec{OD}|} = \frac{|\vec{a} + 2\vec{b}|/5}{|(\vec{a} + 2\vec{b})/3|} = \frac{3}{5}$$

$$\Rightarrow \frac{OP}{OD - OP} = \frac{3}{5 - 3} \Rightarrow \frac{OP}{OD} = \frac{3}{2}$$

10. Determine the value of c so that for all real x , the

vectors $cx\vec{i} - 6\vec{j} + 3\vec{k}$ and $x\vec{i} + 2\vec{j} + 2cx\vec{k}$ make an obtuse angle with each other.

Soln.

Since the angle between the vectors $cx\vec{i} - 6\vec{j} + 3\vec{k}$

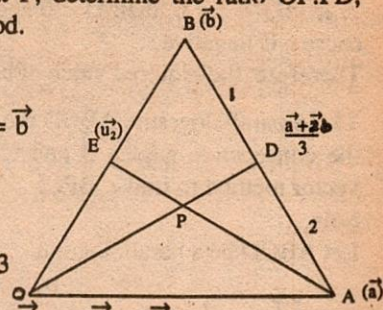
and $x\vec{i} + 2\vec{j} + 2cx\vec{k}$ is obtuse ($>90^\circ$), hence we have

$$\frac{(cx\vec{i} - 6\vec{j} + 3\vec{k}) \cdot (x\vec{i} + 2\vec{j} + 2cx\vec{k})}{\sqrt{c^2x^2 + 6^2 + 3^2} \sqrt{x^2 + 2^2 + 2^2c^2x^2}} = \cos \theta < 0$$

$$\Rightarrow cx^2 - 12 + 6cx < 0$$

$$\Rightarrow cx^2 + 6cx - 12 < 0 \quad \dots\dots\dots(1)$$

$$\text{Now, } D = 36c^2 + 4 \cdot c \cdot 12 = 36c(c + 4/3)$$



∴ if $-4/3 < c < 0$, then $D < 0$ and (1) is satisfied because c is negative.

Therefore, the required value of $c \in [-4/3, 0]$.

11. In parallelogram ABCD the interior bisectors of the consecutive angles B and C intersect at P. Use vector method to find $\angle BPC$.

Soln.

Let ABCD be a parallelogram.

Let $\vec{BA} = \vec{a}$ and

$\vec{BC} = \vec{c}$

Then the equation of the bisector BP is given by

$$\vec{BP} = r [\vec{a}/BA + \vec{c}/BC] \quad \dots\dots\dots(1)$$

And the equation of the bisector CP is given by

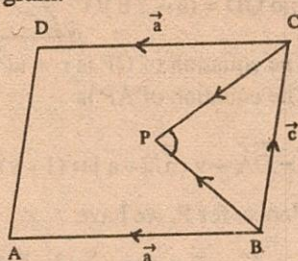
$$\vec{CP} = v [\vec{a}/CD - \vec{c}/CB] = v [\vec{a}/BA - \vec{c}/BC] \quad \dots\dots\dots(2)$$

$$\therefore \vec{BP} \cdot \vec{CP} = uv [\vec{a}/BA + \vec{c}/BC] \cdot [\vec{a}/BA - \vec{c}/BC]$$

$$= uv [|\vec{a}|^2/BA^2 - |\vec{c}|^2/BC^2]$$

$$= uv [BA^2/BA^2 - BC^2/BC^2] = uv (1 - 1) = 0$$

$$\Rightarrow \angle BPC = 90^\circ$$



But the bisector is (given)

$$\vec{r} = -\vec{i} + \vec{j} - \vec{k} \quad \dots\dots\dots(4)$$

Hence, we have

$$-\vec{i} + \vec{j} - \vec{k} = \frac{t(5x+3)\vec{i} + t(5y+4)\vec{j} + 5zt\vec{k}}{5} \quad \dots\dots\dots(5)$$

Since, $\vec{i}, \vec{j}, \vec{k}$ are non-coplanar vectors, hence we have $t(5x+3) = -5$ $\dots\dots\dots(6)$

$$t(5y+4) = 5 \quad \dots\dots\dots(7)$$

$$5zt = -5 \quad \dots\dots\dots(8)$$

$$(6) \Rightarrow 5x+3 = -5/t$$

$$\Rightarrow 5x = -5/t - 3 = -(5+3t)/t$$

$$\Rightarrow x = -(5+3t)/5t$$

$$\text{Similarly, (7)} \Rightarrow y = (5-4t)/5t$$

$$\text{and (8)} \Rightarrow z = -1/t$$

Putting these values of x, y, z in (2), we get,

$$(5+3t)^2/25t^2 + (5-4t)^2/25t^2 + 1/t^2 = 1$$

$$\Rightarrow 25 + 30t + 9t^2 + 25 - 40t + 16t^2 + 25 = 25t^2$$

$$\Rightarrow 25t^2 - 10t + 75 = 25t^2 \Rightarrow 10t = 75 \Rightarrow t = 15/2$$

$$\therefore x = \frac{-(5+3 \cdot 15/2)}{5 \cdot 15/2} = \frac{-(10+45)}{5 \cdot 15} = \frac{-55}{5 \cdot 15} = \frac{-11}{15}$$

$$y = -10/15 \text{ and } z = -2/15 \Rightarrow$$

$$\hat{c} = \frac{1}{15} [-11\vec{i} - 10\vec{j} - 2\vec{k}] = -\frac{1}{15} [11\vec{i} + 10\vec{j} + 2\vec{k}]$$

13. Find the distance of the point $B(\vec{i} + 2\vec{j} + 3\vec{k})$

from the line which is passing through $A(4\vec{i} + 6\vec{j} + 2\vec{k})$ and which is parallel to the vector $2\vec{i} + 3\vec{j} + 6\vec{k}$.

Soln.

$$\vec{AB} = \text{p.v. of B} - \text{p.v. of A}$$

$$= \vec{i} + 2\vec{j} + 3\vec{k}$$

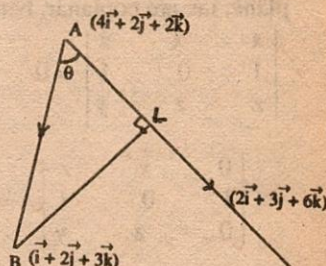
$$4\vec{i} - 2\vec{j} - 2\vec{k} =$$

$$-3\vec{i} + \vec{k}$$

Let BL be $\perp$ on the line through A parallel to the

vector $2\vec{i} + 3\vec{j} + 6\vec{k}$

Let $\angle BAL = \theta$ then,



12. The vector $-\vec{i} + \vec{j} - \vec{k}$ bisects the angle between the vector $\vec{c}$ and $3\vec{i} + 4\vec{j}$. Determine the unit vector $\hat{c}$.

Soln.

$$\text{Let } \hat{c} = x\vec{i} + y\vec{j} + z\vec{k} \quad \dots\dots\dots(1)$$

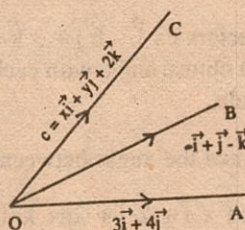
$$\text{Where, } x^2 + y^2 + z^2 = 1 \quad \dots\dots\dots(2)$$

The equation of the internal bisector of

$3\vec{i} + 4\vec{j}$ and $\hat{c}$ is

$$\vec{r} = t \left[x\vec{i} + y\vec{j} + z\vec{k} + \frac{3\vec{i} + 4\vec{j}}{5} \right]$$

$$= \frac{(5xt+3t)\vec{i} + (5yt+4t)\vec{j} + 5zt\vec{k}}{5} \quad \dots\dots\dots(3)$$



$$\cos \theta = \frac{(2\vec{i} + 3\vec{j} + 6\vec{k}) \cdot (-3\vec{i} + \vec{k})}{\sqrt{4+9+36} \sqrt{9+1}} = \frac{-6+6}{7\sqrt{10}} = 0$$

$\Rightarrow$ L coincides with A

$\therefore$ distance of AL from B

$$AB = |\vec{AB}| = |-3\vec{i} + \vec{k}| = \sqrt{10}.$$

14. Prove that $\{[(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} + \vec{C})\} \cdot (\vec{B} + \vec{C}) = 0$ where $|\vec{B}| = |\vec{C}| = 1$.

Soln.

$$\begin{aligned} \text{We have, } & [(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} + \vec{C}) \\ \Rightarrow & [(\vec{A} + \vec{B}) \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{C}) - [(\vec{A} + \vec{C}) \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{B}) \\ = & [\vec{A} \cdot (\vec{B} \times \vec{C}) + \vec{B} \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{C}) \\ & - [\vec{A} \cdot (\vec{B} \times \vec{C}) + \vec{C} \cdot (\vec{B} \times \vec{C})] (\vec{A} + \vec{B}) \\ = & ([\vec{A} \cdot \vec{B} \cdot \vec{C}] + [\vec{B} \cdot \vec{B} \cdot \vec{C}]) (\vec{A} + \vec{C}) \\ & - ([\vec{A} \cdot \vec{B} \cdot \vec{C}] + [\vec{C} \cdot \vec{B} \cdot \vec{C}]) (\vec{A} + \vec{B}) \\ = & [\vec{A} \cdot \vec{B} \cdot \vec{C}] (\vec{A} + \vec{C}) - [\vec{A} \cdot \vec{B} \cdot \vec{C}] (\vec{A} + \vec{B}) \\ = & [\vec{A} \cdot \vec{B} \cdot \vec{C}] (\vec{A} + \vec{C} - \vec{A} - \vec{B}) = [\vec{A} \cdot \vec{B} \cdot \vec{C}] (\vec{C} - \vec{B}) \\ \therefore & [(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} + \vec{C}) \cdot (\vec{B} + \vec{C}) \\ = & [\vec{A} \cdot \vec{B} \cdot \vec{C}] (\vec{C} - \vec{B}) \cdot (\vec{B} + \vec{C}) \\ = & [\vec{A} \cdot \vec{B} \cdot \vec{C}] (\vec{C} \cdot \vec{B} + \vec{C} \cdot \vec{C} - \vec{B} \cdot \vec{B} - \vec{B} \cdot \vec{C}) \\ = & [\vec{A} \cdot \vec{B} \cdot \vec{C}] [|\vec{C}|^2 - |\vec{B}|^2] = [\vec{A} \cdot \vec{B} \cdot \vec{C}] (1 - 1) = 0. \end{aligned}$$

15. If the vectors $\vec{b}, \vec{c}, \vec{d}$ are not coplanar, then prove that

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$$

is parallel to $\vec{a}$.

Soln.

$$\begin{aligned} \text{1st term} &= (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) \\ &= (\vec{a} \times \vec{b}) \times \vec{l}, \text{ where } \vec{l} = \vec{c} \times \vec{d} \\ &= (\vec{a} \cdot \vec{l}) \vec{b} - (\vec{b} \cdot \vec{l}) \vec{a} \\ &= [\vec{a} \cdot (\vec{c} \times \vec{d})] \vec{b} - [\vec{b} \cdot (\vec{c} \times \vec{d})] \vec{a} \end{aligned}$$

$$\begin{aligned} &= [\vec{a} \cdot \vec{c} \cdot \vec{d}] \vec{b} - [\vec{b} \cdot \vec{c} \cdot \vec{d}] \vec{a} \\ &= [\vec{c} \cdot \vec{a} \cdot \vec{d}] \vec{b} - [\vec{b} \cdot \vec{c} \cdot \vec{d}] \vec{a} \end{aligned} \quad \dots\dots(1)$$

$$\begin{aligned} \text{2nd term} &= (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) \\ &= \vec{m} \times (\vec{d} \times \vec{b}), \text{ where } \vec{m} = \vec{a} \times \vec{c} \\ &= (\vec{m} \cdot \vec{b}) \vec{d} - (\vec{m} \cdot \vec{d}) \vec{b} \\ &= (\vec{a} \times \vec{c} \cdot \vec{b}) \vec{d} - \{(\vec{a} \times \vec{c}) \cdot \vec{d}\} \vec{b} \\ &= [\vec{a} \cdot \vec{c} \cdot \vec{b}] \vec{d} - [\vec{a} \cdot \vec{c} \cdot \vec{d}] \vec{b} \\ &= -[\vec{a} \cdot \vec{b} \cdot \vec{c}] \vec{d} + [\vec{c} \cdot \vec{a} \cdot \vec{d}] \vec{b} \end{aligned} \quad \dots\dots(2)$$

$$\begin{aligned} \text{3rd term} &= (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c}) \\ &= (\vec{a} \times \vec{d}) \times \vec{n}, \text{ where } \vec{n} = \vec{b} \times \vec{c} \\ &= (\vec{a} \cdot \vec{n}) \vec{d} - (\vec{d} \cdot \vec{n}) \vec{a} \\ &= \{\vec{a} \cdot (\vec{b} \times \vec{c})\} \vec{d} - \{\vec{d} \cdot (\vec{b} \times \vec{c})\} \vec{a} \\ &= [\vec{a} \cdot \vec{b} \cdot \vec{c}] \vec{d} - [\vec{d} \cdot \vec{b} \cdot \vec{c}] \vec{a} \\ &= [\vec{a} \cdot \vec{b} \cdot \vec{c}] \vec{d} - [\vec{b} \cdot \vec{c} \cdot \vec{d}] \vec{a} \end{aligned} \quad \dots\dots(3)$$

Adding (1), (2) and (3), we get,

$$\begin{aligned} \text{LHS} &= [\vec{c} \cdot \vec{a} \cdot \vec{d}] \vec{b} - [\vec{b} \cdot \vec{c} \cdot \vec{d}] \vec{a} - [\vec{a} \cdot \vec{b} \cdot \vec{c}] \vec{d} + [\vec{c} \cdot \vec{a} \cdot \vec{d}] \vec{b} \\ &\quad + [\vec{a} \cdot \vec{b} \cdot \vec{c}] \vec{d} - [\vec{b} \cdot \vec{c} \cdot \vec{d}] \vec{a} \\ &= -2 [\vec{b} \cdot \vec{c} \cdot \vec{d}] \vec{a} \end{aligned} \quad \dots\dots(4)$$

Since $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar vectors, hence

$[\vec{b} \cdot \vec{c} \cdot \vec{d}] \neq 0$. Therefore (4) shows that the LHS is a vector parallel to $\vec{a}$.

16. If $\vec{A} = (1, a, a^2)$, $\vec{B} = (1, b, b^2)$, $\vec{C} = (1, c, c^2)$ are non-equal coplanar

$$\text{and } \begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$$

then show that $abc = -1$

Soln.

Since the vectors $\vec{A} = (1, a, a^2)$, $\vec{B} = (1, b, b^2)$ and

$\vec{c} = (1, c, c^2)$ are non-coplanar vectors, hence

$$\Rightarrow [\vec{a}, \vec{b}, \vec{c}] \neq 0$$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \quad \dots\dots(1)$$

Now, $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow 1 + abc = 0 \quad \therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$$\Rightarrow abc = -1.$$

17. If the vectors $a\vec{i} + \vec{j} + \vec{k}$, $\vec{i} + b\vec{j} + \vec{k}$ and $\vec{i} + \vec{j} + c\vec{k}$ ($a \neq 1$, $b \neq 1$, $c \neq 1$) are coplanar, then find the value of $1/(1-a) + 1/(1-b) + 1/(1-c)$.

Soln.

Since the vectors $a\vec{i} + \vec{j} + \vec{k}$, $\vec{i} + b\vec{j} + \vec{k}$, $\vec{i} + \vec{j} + c\vec{k}$ are coplanar, hence

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a-1 & 0 & 1-c \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\Rightarrow (a-1)[(b-1)c - (1-c)] + (1-c)[0 - (b-1)] = 0$$

$$\Rightarrow (a-1)(b-1)c + (a-1)(c-1) + (c-1)(b-1) = 0$$

Dividing by $(a-1)(b-1)(c-1)$, we get

$$\frac{c}{(c-1)} + \frac{1}{(b-1)} + \frac{1}{(a-1)} = 0$$

$$\Rightarrow \frac{[(c-1)+1]}{(c-1)} + \frac{1}{(b-1)} + \frac{1}{(a-1)} = 0$$

$$\Rightarrow 1 + \frac{1}{(c-1)} + \frac{1}{(b-1)} + \frac{1}{(a-1)} = 0$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1.$$

18. Let $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$ and

$\vec{c} = \vec{i} + \vec{j} - 2\vec{k}$ be three vectors. A vector in the plane

of $\vec{b}$ and $\vec{c}$ whose projections on $\vec{a}$ is of magnitude $\sqrt{2/3}$ is

Soln.

Let $\vec{d}$ be a vector in the plane of $\vec{b}$ and $\vec{c}$ so that we

have, $\vec{d} = \vec{b} + x\vec{c}$

$$= (\vec{i} + 2\vec{j} - \vec{k}) + x(\vec{i} + \vec{j} - 2\vec{k})$$

$$= (1+x)\vec{i} + (2+x)\vec{j} - (1+2x)\vec{k}$$

$$\therefore \sqrt{\frac{2}{3}} = \text{projection of } \vec{d} \text{ on } \vec{a}$$

$$= \pm \vec{d} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$\Rightarrow \pm [(1+x)\vec{i} + (2+x)\vec{j} - (1+2x)\vec{k}] \cdot \frac{2\vec{i} - \vec{j} + \vec{k}}{\sqrt{6}}$$

$$= \pm \frac{2(1+x) - (2+x) - (1+2x)}{\sqrt{6}} = \pm \frac{-1-x}{\sqrt{6}}$$

$$\Rightarrow 1+x = \pm 2$$

$$\Rightarrow x = \pm 2 - 1 = 1 \text{ or } -3$$

$$\text{when } x = 1, \vec{d} = 2\vec{i} + 3\vec{j} - 3\vec{k}$$

$$\text{and when } x = -3, \vec{d} = -2\vec{i} - \vec{j} + 5\vec{k}.$$

19. Let $\vec{a} = 4\vec{i} + 3\vec{j}$, and $\vec{b}$ be two vectors perpendicular to each other in x-y plane. All the vectors in the same plane having projections 1 and 2 along

$\vec{b}$ and $\vec{c}$ respectively are given by

Soln.

Since $\vec{b}$ is $\perp$ to $\vec{a} = 4\vec{i} + 3\vec{j}$, hence we can take

$$\vec{b} = u(3\vec{i} - 4\vec{j})$$

Let $\vec{R} = x\vec{i} + y\vec{j}$ be the required vector, we have

$$1 = \vec{R} \cdot \hat{\vec{a}} = (x\vec{i} + y\vec{j}) \cdot \frac{4\vec{i} + 3\vec{j}}{5} \Rightarrow 4x + 3y = 5 \quad \dots\dots(1)$$

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$$\text{And, } 2 = \vec{R} \cdot \hat{b} = (\vec{x}\vec{i} + \vec{y}\vec{j}) \cdot \frac{u(3\vec{i} - 4\vec{j})}{\pm 5u}$$

$$= \pm \frac{3x - 4y}{5}, \quad \dots\dots(2)$$

$\pm$ sign are taken as u may have positive or negative value.

$$\Rightarrow 3x - 4y = \pm 10$$

Solving (1) and (2), we get

$$x = -\frac{2}{5} \text{ and } y = \frac{11}{5} \text{ or } x = 2, y = -1$$

$$\therefore \vec{R} = \frac{-2\vec{i} + 11\vec{j}}{5} \text{ or } 2\vec{i} - \vec{j}.$$

20. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\vec{i}, \vec{i} + \vec{j}$ and the plane determined by the vectors

$\vec{i} - \vec{j}, \vec{j} + \vec{k}$. The angle between $\vec{a}$ and the vector $\vec{i} - 2\vec{j} + 2\vec{k}$ is

Soln.

A vector $\perp$ to the plane determined by the vectors

$\vec{i}$ and $\vec{i} + \vec{j}$ is

$$\vec{i} \times (\vec{i} + \vec{j}) = \vec{i} \times \vec{i} + \vec{i} \times \vec{j} = \vec{k};$$

and a vector $\perp$ to the plane determined by the vectors

$\vec{i} - \vec{j}$ and $\vec{i} + \vec{k}$ is

$$(\vec{i} - \vec{j}) \times (\vec{i} + \vec{k}) = \vec{i} \times \vec{i} + \vec{i} \times \vec{k} - \vec{j} \times \vec{i} - \vec{j} \times \vec{k}$$

$$= -\vec{j} + \vec{k} - \vec{i} = -\vec{i} - \vec{j} + \vec{k}$$

$\vec{a}$ is parallel to the vector $\vec{k} \times (-\vec{i} - \vec{j} + \vec{k})$

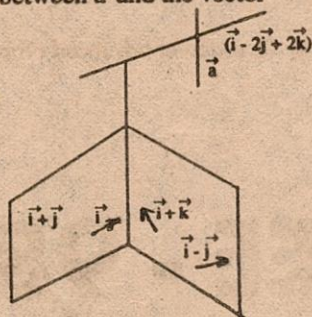
$$= -\vec{k} \times \vec{i} - \vec{k} \times \vec{j} + \vec{k} \times \vec{k} = -\vec{j} + \vec{i} = \vec{i} - \vec{j}$$

If θ be the required angle, then

$$\cos \theta = \frac{(\vec{i} - \vec{j}) \cdot (\vec{i} - 2\vec{j} + 2\vec{k})}{\sqrt{(1^2 + 1^2)} \sqrt{(1^2 + 2^2 + 2^2)}}$$

$$= \frac{1 + 2}{\sqrt{2} \sqrt{9}} = \frac{3}{\sqrt{2} \cdot 3} = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

$$\Rightarrow \theta = 45^\circ$$



21. A, B, C and D are four points such that

$$\vec{AB} = m(2\vec{i} - 6\vec{j} + 2\vec{k}), \vec{BC} = \vec{i} - 2\vec{j} \text{ and } \vec{CD} =$$

$n(-6\vec{i} + 15\vec{j} - 3\vec{k})$. Find the conditions on the scalars m and n so that CD intersect AB at point E . Also find the area of the ΔCBE .

Soln.

Since AB and CD intersect at E , hence $EB < AB$ and $CE < CD$

$$\therefore \vec{EB} = p \vec{AB}, \quad 0 < p \leq 1$$

$$\vec{CE} = q \vec{CD}, \quad 0 < q \leq 1$$

$$\text{In } \Delta EBC, \vec{EB} + \vec{BC} + \vec{CE} = 0$$

$$\Rightarrow p \vec{AB} + \vec{BC} + q \vec{CD} = 0$$

$$\Rightarrow pm(2\vec{i} - 6\vec{j} + 2\vec{k}) + (\vec{i} - 2\vec{j})$$

$$+ qn(-6\vec{i} + 15\vec{j} - 3\vec{k}) = 0$$

$$\Rightarrow (2pm + 1 - 6qn)\vec{i} - (6pm + 2 - 15qn)\vec{j}$$

$$+ (2pm - 3qn)\vec{k} = 0$$

Since, $\vec{i}, \vec{j}, \vec{k}$ are non-coplanar vectors, hence we have

$$2pm + 1 - 6qn = 0 \quad \dots\dots(1)$$

$$6pm + 2 - 15qn = 0 \quad \dots\dots(2)$$

$$\text{and } 2pm - 3qn = 0 \quad \dots\dots(3)$$

Solving for pm and qn , we get

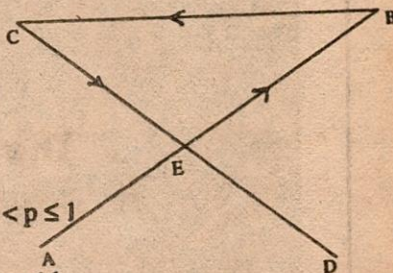
$$pm = \frac{1}{2}, qn = \frac{1}{3} \Rightarrow p = \frac{1}{2m}, q = \frac{1}{3n}$$

$$\therefore 0 < p \leq 1 \Rightarrow 0 < \frac{1}{2m} \leq 1 \Rightarrow m \geq \frac{1}{2}$$

$$\text{and } 0 < q \leq 1 \Rightarrow 0 < \frac{1}{3n} \leq 1 \Rightarrow n \geq \frac{1}{3}$$

$$\text{Further, area of } BCE = \frac{1}{2} |\vec{EB} \times \vec{EC}|$$

$$= \frac{1}{2} |p \vec{AB} \times -q \vec{CD}| = \frac{1}{2} pq |\vec{AB} \times \vec{CD}|$$



$$= \frac{1}{2} p q m n \left| \begin{vmatrix} i & j & k \\ -6 & 15 & -3 \\ 2 & -6 & 2 \end{vmatrix} \right|$$

$$= \frac{1}{2} p q m n [(30 - 18)\vec{i} + (-6 + 12)\vec{j} + (36 - 30)\vec{k}]$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3} |12\vec{i} + 6\vec{j} + 6\vec{k}|$$

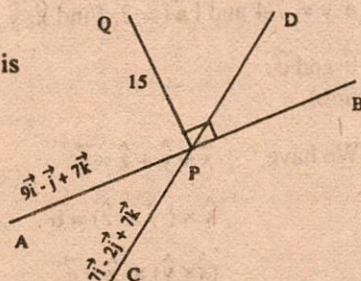
$$= \frac{1}{12} \cdot 6 \sqrt{2^2 + 1^2 + 1^2} = \frac{\sqrt{6}}{2}$$

22. The position vectors of two points A and C are $9\vec{i} - \vec{j} + 7\vec{k}$ and $7\vec{i} - 2\vec{j} + 7\vec{k}$ respectively. The point of intersection of vectors $\vec{AB} = 4\vec{i} - \vec{j} + 3\vec{k}$ and

$\vec{CD} = 2\vec{i} - \vec{j} + 2\vec{k}$ is P. If vector PQ is perpendicular to AB and CD and PQ = 15 units, find the position vector of Q.

Soln.

The equation of AP is



$$\vec{r} = \text{p.v. of A} + \vec{AP}$$

$$= 9\vec{i} - \vec{j} + 7\vec{k} + u\vec{AB}$$

$$= 9\vec{i} - \vec{j} + 7\vec{k} + u(4\vec{i} - \vec{j} + 3\vec{k})$$

$$= (9 + 4u)\vec{i} - (1 + u)\vec{j} + (7 + 3u)\vec{k} \quad \text{.....(1)}$$

The equation of CP is

$$\vec{r} = \text{p.v. of C} + \vec{CP}$$

$$= 7\vec{i} - 2\vec{j} + 7\vec{k} + v\vec{CD}$$

$$= 7\vec{i} - 2\vec{j} + 7\vec{k} + v(2\vec{i} - \vec{j} + 2\vec{k})$$

$$= (7 + 2v)\vec{i} - (2 + v)\vec{j} + (7 + 2v)\vec{k} \quad \text{.....(2)}$$

For the point P, we have

$$(9 + 4u)\vec{i} - (1 + u)\vec{j} + (7 + 3u)\vec{k} =$$

$$(7 + 2v)\vec{i} - (2 + v)\vec{j} + (7 + 2v)\vec{k}$$

Since $\vec{i}, \vec{j}, \vec{k}$ are non-coplanar, hence we have

$$9 + 4u = 7 + 2v$$

$$\Rightarrow 4u - 2v + 2 = 0$$

$$\Rightarrow 2u - v + 1 = 0; \quad \text{.....(3)}$$

$$1 + u = 2 + v \Rightarrow u - v - 1 = 0; \quad \text{.....(4)}$$

$$\text{and } 7 + 3u = 7 + 2v \Rightarrow 3u = 2v \quad \text{.....(5)}$$

Solving (3) and (4), we get

$$u = -2, v = -3$$

These values also satisfy (5)

Hence, the position vector of P is

$$\vec{r} = \vec{i} + \vec{j} + \vec{k} \quad \text{.....(6)}$$

Also, a vector $\perp$ to AB and CD is

$$\vec{AB} \times \vec{CD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 3 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= (-2 + 3)\vec{i} - (6 - 8)\vec{j} + (-4 + 2)\vec{k}$$

$$= \vec{i} + 2\vec{j} - 2\vec{k}$$

$$PQ = 15 \cdot \frac{|\vec{i} + 2\vec{j} - 2\vec{k}|}{\sqrt{1^2 + 2^2 + 2^2}} = 5(\vec{i} + 2\vec{j} - 2\vec{k})$$

Hence, p.v. of Q = p.v. of P + $\vec{PQ}$

$$= \vec{i} + \vec{j} + \vec{k} + 5(\vec{i} + 2\vec{j} - 2\vec{k})$$

$$= 6\vec{i} + 11\vec{j} - 9\vec{k}$$

23. The position vectors of the vertices A, B and C

of a tetrahedron ABCD are $\vec{i} + \vec{j} + \vec{k}$, $\vec{i}$ and $3\vec{i}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line through A of the angle BAC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is $2\sqrt{2}/3$, find the position vector of the point E for all its possible positions.

Soln.

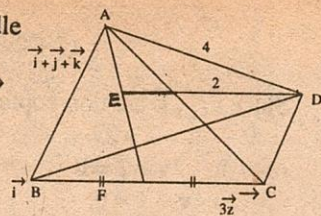
The position vector of F, the middle

$$\text{point of BC is } \frac{\vec{i} + 3\vec{i}}{2} = 2\vec{i}$$

tion vector of F, the middle

$$\text{point of BC is } \frac{\vec{i} + 3\vec{i}}{2} = 2\vec{i}$$

$$\therefore \vec{AF} = \text{p.v. of F}$$



$$\text{p.v. of A} = 2\vec{i} - (\vec{i} + \vec{j} + \vec{k})$$

$$= \vec{i} - \vec{j} - \vec{k}$$

$$\text{Volume of the tetrahedron ABCD} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \frac{1}{3} \Delta ABC \cdot ED = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \frac{1}{2} |\vec{BC} \times \vec{BA}| \cdot ED = 2\sqrt{2}$$

$$\Rightarrow |2\vec{i} \times (\vec{j} + \vec{k})| \cdot ED = 4\sqrt{2}$$

$$\Rightarrow 2|\vec{i} \times \vec{j} + \vec{i} \times \vec{k}| \cdot ED = 4\sqrt{2}$$

$$\Rightarrow |\vec{k} - \vec{j}| \cdot ED = 2\sqrt{2} \Rightarrow \sqrt{1} \cdot ED = 2\sqrt{2}$$

$$\Rightarrow ED = 2$$

$$AE^2 = AD^2 - ED^2 = 16 - 4 = 12 \Rightarrow AE = 2\sqrt{3}$$

$$\therefore \vec{AE} = 2\sqrt{3} \frac{\vec{i} - \vec{j} - \vec{k}}{\pm \sqrt{3}} = \pm 2(\vec{i} - \vec{j} - \vec{k})$$

$$\text{p.v. of E} = \text{p.v. of A} + \vec{AE}$$

$$= \vec{i} + \vec{j} + \vec{k} \pm 2(\vec{i} - \vec{j} - \vec{k})$$

$$= 3\vec{i} - \vec{j} - \vec{k} \text{ or } -\vec{i} + 3\vec{j} + 3\vec{k}$$

24. Let $\hat{a}$ be a unit vector and $\vec{b}$ be a non-equal zero vector not parallel to $\hat{a}$. Find the angles of the triangle, two sides of which are represented by the vectors $3(\hat{a} \times \vec{b})$ and $\vec{b} - (\hat{a} \cdot \vec{b})\hat{a}$.

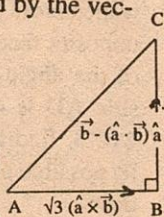
Soln.

$$\text{Let } \vec{AB} = 3(\hat{a} \times \vec{b}) \text{ and } \vec{BC} = \vec{b} - (\hat{a} \cdot \vec{b})\hat{a}$$

$$\text{Then, } \vec{AB} \cdot \vec{BC} = \sqrt{3}(\hat{a} \times \vec{b}) \cdot \{\vec{b} - (\hat{a} \cdot \vec{b})\hat{a}\}$$

$$= \sqrt{3} \{(\hat{a} \times \vec{b}) \cdot \vec{b}\} - \sqrt{3}(\hat{a} \cdot \vec{b}) \{(\hat{a} \times \vec{b}) \cdot \hat{a}\}$$

$$= \sqrt{3} [\vec{a} \cdot \vec{b} \cdot \vec{b}] - \sqrt{3}(\hat{a} \cdot \vec{b}) [\hat{a} \cdot \vec{b} \cdot \hat{a}]$$



$$= 0 - 0 = 0$$

$$\Rightarrow AB \perp BC \text{ i.e. } B = 90^\circ$$

$$\text{Now, } AB^2 = 3(\hat{a} \times \vec{b})^2 = 3(1^2 \cdot b^2 \sin^2 \theta) = 3b^2 \sin^2 \theta \quad \dots\dots(1)$$

$$\text{and } BC^2 = [\vec{b} - (\hat{a} \cdot \vec{b})\hat{a}]^2$$

$$= b^2 + (\hat{a} \cdot \vec{b})^2 - 2(\hat{a} \cdot \vec{b})^2 = b^2 - b^2 \cos^2 \theta = b^2(1 - \cos^2 \theta) = b^2 \sin^2 \theta \quad \dots\dots(2)$$

From (1) and (2), we have

$$AB^2 = 3 BC^2 \Rightarrow AB = \sqrt{3} BC$$

$$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}} \Rightarrow \tan A = \tan 30^\circ$$

$$\Rightarrow A = 30^\circ \Rightarrow \therefore C = 60^\circ$$

25. Let $\hat{x}$, $\hat{y}$ and $\hat{z}$ be unit vectors such that $\hat{x} + \hat{y} + \hat{z}$

$$= \hat{a}, \hat{x} \times (\hat{y} \times \hat{z}) = \vec{b}, (\hat{x} \times \hat{y}) \times \hat{z} = \vec{c}, \hat{a} \cdot \hat{x} = 3/2,$$

$$\hat{a} \cdot \hat{y} = 7/4 \text{ and } |\hat{a}| = 2. \text{ Find } \hat{x}, \hat{y} \text{ and } \hat{z} \text{ in terms of } \hat{a},$$

$$\vec{b} \text{ and } \vec{c}.$$

Soln.

$$\text{We have } \hat{x} + \hat{y} + \hat{z} = \vec{a} \quad \dots\dots(1)$$

$$\hat{x} \times (\hat{y} \times \hat{z}) = \vec{b} \quad \dots\dots(2)$$

$$(\hat{x} \times \hat{y}) \times \hat{z} = \vec{c} \quad \dots\dots(3)$$

$$\hat{a} \cdot \hat{x} = \frac{3}{2}, \hat{a} \cdot \hat{y} = \frac{7}{4} \text{ and } |\hat{a}| = 2 \quad \dots\dots(4)$$

Taking dot product of both sides of (1) with $\vec{a}$, we get

$$\hat{a} \cdot \hat{x} + \hat{a} \cdot \hat{y} + \hat{a} \cdot \hat{z} = \hat{a} \cdot \vec{a} = |\hat{a}|^2 = 4$$

$$\Rightarrow \frac{3}{2} + \frac{7}{4} + \hat{a} \cdot \hat{z} = 4 \Rightarrow \hat{a} \cdot \hat{z} = \frac{3}{4} \quad \dots\dots(5)$$

Taking dot product of (1) with $\hat{x}$, $\hat{y}$ and $\hat{z}$ successively, we get,

$$\hat{x} \cdot \hat{x} + \hat{x} \cdot \hat{y} + \hat{x} \cdot \hat{z} = \hat{a} \cdot \hat{x}$$

$$\Rightarrow 1 + \hat{x} \cdot \hat{y} + \hat{x} \cdot \hat{z} = \frac{3}{2} \quad \dots\dots(6)$$

$$\hat{x} \cdot \hat{y} + \hat{y} \cdot \hat{y} + \hat{y} \cdot \hat{z} = \hat{a} \cdot \hat{y}$$

$$\Rightarrow \hat{x} \cdot \hat{y} + 1 + \hat{y} \cdot \hat{z} = \frac{7}{4} \quad \dots\dots(7)$$

$$\text{and } \hat{z} \cdot \hat{x} + \hat{y} \cdot \hat{z} + \hat{z} \cdot \hat{z} = \hat{a} \cdot \hat{z}$$

$$\Rightarrow \hat{z} \cdot \hat{x} + \hat{y} \cdot \hat{z} + 1 = \frac{3}{4} \quad \dots\dots(8)$$

Adding (6), (7) and (8), we get

$$3 + 2(\hat{x} \cdot \hat{y} + \hat{y} \cdot \hat{z} + \hat{z} \cdot \hat{x}) = \frac{3}{2} + \frac{7}{4} + \frac{3}{4} = 4$$

$$\Rightarrow \hat{x} \cdot \hat{y} + \hat{y} \cdot \hat{z} + \hat{z} \cdot \hat{x} = \frac{(4-3)}{2} = \frac{1}{2} \quad \dots\dots(9)$$

Now,

$$(9) - (8) \Rightarrow \hat{x} \cdot \hat{y} - 1 = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$$

$$\Rightarrow \hat{x} \cdot \hat{y} = 1 - \frac{1}{4} = \frac{3}{4} \quad \dots\dots(10)$$

$$(9) - (6) \Rightarrow \hat{y} \cdot \hat{z} - 1 = \frac{1}{2} - \frac{3}{2} = -1$$

$$\Rightarrow \hat{y} \cdot \hat{z} = 0 \quad \dots\dots(11)$$

$$\text{and } (9) - (7) \Rightarrow \hat{z} \cdot \hat{x} - 1 = \frac{1}{2} - \frac{7}{4} = -\frac{5}{4}$$

$$\Rightarrow \hat{z} \cdot \hat{x} = 1 - \frac{5}{4} = -\frac{1}{4} \quad \dots\dots(12)$$

$$(2) \Rightarrow (\hat{x} \cdot \hat{z}) \hat{y} - (\hat{x} \cdot \hat{y}) \hat{z} = \vec{b}$$

$$\Rightarrow \left(-\frac{1}{4}\right) \hat{y} - \left(\frac{3}{4}\right) \hat{z} = \vec{b} \quad \dots\dots(13)$$

$$\text{and } (3) \Rightarrow (\hat{x} \cdot \hat{z}) \hat{y} - (\hat{y} \cdot \hat{z}) \hat{x} = \vec{c}$$

$$\Rightarrow \left(-\frac{1}{4}\right) \hat{y} = \vec{c}$$

$$\Rightarrow \hat{y} = -4\vec{c} \quad \dots\dots(14)$$

$$\therefore (13) \text{ gives } \left(\frac{3}{4}\right) \hat{z} = \vec{c} - \vec{b}$$

$$\Rightarrow \hat{z} = \left(\frac{4}{3}\right) (\vec{c} - \vec{b}) \quad \dots\dots(15)$$

$$(1) \Rightarrow \hat{x} = \vec{a} - \hat{y} - \hat{z} = \vec{a} + 4\vec{c} - \left(\frac{4}{3}\right) (\vec{c} - \vec{b})$$

$$= \vec{a} + \left(\frac{4}{3}\right) \vec{b} + \left(\frac{8}{3}\right) \vec{c} \quad \dots\dots(16)$$

$$\text{Hence, } \hat{x} = \vec{a} + \frac{4}{3} \vec{b} + \frac{8}{3} \vec{c}$$

$$\hat{y} = -4\vec{c} \text{ and } \hat{z} = \frac{4}{3} (\vec{c} - \vec{b}).$$

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RUSH BEFORE THE STOCKS LAST

HISTORY OF MATHEMATICS

EUCLID

Euclid was born in the late fourth century B.C., by that time Greek geometry had come a long way from the state in which Pythagoras had left it. It had become more scientific and less mystical. Scores of new theorems had been added; curves, circles, and solids were studied as well as straight lines and plane figures.

The best-known school for mathematics and the closely related subject of philosophy was Plato's Academy. Famous and respected, the Academy was the Oxford, Cambridge, Yale, and Harvard universities of its time all rolled into one. Here the golden flowering of Greece occurred, and here the greatest minds of antiquity assembled to lecture and listen. Established in 380 B.C., it survived repeated invasions, outlived tyrant after tyrant, and saw two great civilizations fall—the Greek and the Roman—before its doors were finally closed in the sixth century A.D. by the Emperor Justinian. Euclid is believed to have studied at the Academy, learning all that Greece had to teach about mathematics.

With Euclid, Greek or Hellenic geometry rose to its highest. He represents the culmination of three centuries of mathematical thought, and his crowning achievement is the *Elements*, thirteen books in which he presents the basic principles of geometry and the statements and proofs of the theorems, systematized into a consistent whole. Euclid himself did not discover or invent—depending on one's viewpoint—all the theorems presented in his *Elements*. He compiled the work of former mathematicians—Pythagoras, Eudoxus, Menaechmus, Hippocrates, etc., improving on the old proofs, giving new and simpler ones where needed and selecting his axioms with care. Different schools of Greek mathematicians often used different

axioms—some that were not really "self-evident." But Euclid selected only the most basic and obvious ones. He was careful to take nothing for granted. Everything was rigorously proved; and the result was a system of cold, logical, objective beauty.

Almost immediately the *Elements* became—and remains—the standard text in geometry. When the printing press was invented, Euclid's *Elements* was among the first books to be printed. It is the all-time best seller as far as textbooks go, and wherever geometry is taught, there is Euclid.

Besides writing his definitive book on geometry, Euclid, like all Greek geometers, tried to solve the three classical problems of trisecting an angle, doubling a cube and squaring a circle. For centuries the Greeks tried to solve these problems, using an unmarked ruler and a string compass.

The first problem seems simple enough and resolves itself into the cubic equation $4x^3 - 3x - a = 0$, where a is a given number. It can be done with certain marked instruments—but not with the unmarked ones the Greeks used. Although the Greeks did not know it, a ruler and compass can only be used to solve problems that resolve themselves into linear and quadratic equations, but not cubic ones. (A linear equation is one in which no term is raised to a power higher than 1; a quadratic equation contains terms raised to the second power, such as x^2 , but no higher; and a cubic has terms raised to the third to the power, such as x^3 .)

The second problem—doubling a cube—also resolves itself into a cubic equation, $2x^3 = y^3$, or even more simply, $x^3 = 2$. This problem, also insoluble with unmarked tools, has a pitfall into which many an amateur falls and which has been celebrated in

legend. The Athenians, according to the story, once consulted the Delian oracle before understanding a military campaign and were told that to insure victory they should double the size of the altar of Apollo, which was a cube. They duly built an altar twice as long, twice as wide, and twice as high as the original. Believing long that they had fulfilled the oracle's request, they went confidently to war--and lost. Actually they had made the altar eight times as large, not two.

The third problem, squaring the circle, is not possible at all with any kind of instrument, nor can it be stated algebraically. The problem involves finding an exact value for π (pi), the ratio between a circle and its diameter. But π is an irrational, transcendental number for which there is no exact value, and anyone who tries to find an exact value is simply wasting his time, for it has been proved that it cannot be done.

Although the Greeks failed in their attempts to solve these three problems, they were led to explore different areas of geometry, mainly conic sections, in their search for solutions.

Euclid also attacked the old problem of irrationals that had given Pythagoras so much trouble and even succeeded in proving their existence. Taking a right triangle whose arms are each 1, he showed that there is no rational solution for the length of the hypotenuse, which is $x^2 = 2$. Euclid first assumed that a solution could be found. The solution would have to be a number in its lowest terms, p/q , such that $p^2/q^2 = 2$. Either p or q or both must be odd, for if they are both even, the fraction can be divided by 2 and is not in its lowest terms. Euclid then showed that p cannot be odd by substituting p/q for x in the original equation, $x^2 = 2$, which gives $p^2/q^2 = 2$. Transposing, we get $p^2 = 2q^2$. Since p^2 equals twice another number, p^2 must be even; and since the square root of an even number is also an even number, p must be even.

Therefore, if a solution is possible, q must be odd.

We already know that p is even and therefore is equivalent to twice another number, or $2r$. Substituting $2r$ for p in the previous equation $p^2 = 2q^2$, we get $4r^2 = 2q^2$, or $2r^2 = q^2$. Therefore q , for the same reasons as stated for p , is even. yet the fraction p/q was assumed to be in its lowest terms. But if both p and q are even, the fraction cannot be in its lowest terms. Thus, nothing but absurd contradictions result from the assumption that the problem can be solved rationally, i.e., with the numbers the Greeks used. No ratio exists between the numbers p and q such that $p^2/q^2 = 2$. For this reason the square root of 2 is called irrational. Irrational simply means that the number has no ratio, not that it is insane.

In number theory, Euclid, like most mathematicians of his day, studied primes, searching for a test to determine whether any given number is a prime or not. Needless to say, he never found it, but he did settle one question about primes: whether or not they are infinite in number.

Any half-observant schoolchild can see that primes, like the atmosphere, tend to thin out the higher we go. From 2 to 50 there are fifteen prime numbers; from 50 to 100, only ten. It seems entirely possible that primes might thin out and eventually disappear completely. Euclid devised an ingenious proof to show that this is not so; the number of primes is infinite.

If the largest prime is n , he reasoned, then there must be another number large than n which can be generated by multiplying 1 times 2 times 3 and so on up to n and then adding 1 to the result. In mathematical symbols this number would be written $n! + 1$. Now if n is the largest prime, this new number is not a prime. If it is not a prime, it must have a divisor smaller than itself. But if it is divided by any number up to and including n , there will always be a remainder of 1, since the number was formed by multiplying all the numbers from 1 to n together and then adding 1. Therefore, its divisor--if it has one--must be greater than n and that divisor itself must

be a prime, which is a prime number greater than n . If $n! + 1$ has no divisors greater than n , then $n! + 1$ itself is a prime greater than n . In either case, a larger prime than n exists. Therefore, the number of primes is infinite, for in the same way a larger prime than $n! + 1$ can be found, and so on ad infinitum.

This proof of the infinity of primes—which also presupposes in infinity of numbers—and the proof of the existence of irrationals did not spur, Euclid and his colleagues on to a fuller investigation of the nature of infinity. On the contrary, they avoided working with infinity and irrationals—which are part and parcel of infinity for an irrational number is itself infinite. Pythagoras had become so upset by irrationals that he tried to suppress them. Euclid, while not as extreme in his reaction, ignored the whole idea of infinity and infinite irrationals, preferring to deal with finite, static shapes that stood still and did not go bounding off into distant space.

The problem of infinity, however, is not one that can be ignored. No matter how much it is written out of the script and shunted off into the wings, it constantly creeps back on stage to shout, "Here I am! What part do I play in mathematics?" For 2,500 years no one knows exactly when or where, but the famous school at Alexandria continued.

THEY SAID IT!

"Mathematicians are like Frenchmen. Whatever you say to them they translate into their own language and forthwith it is something entirely different."

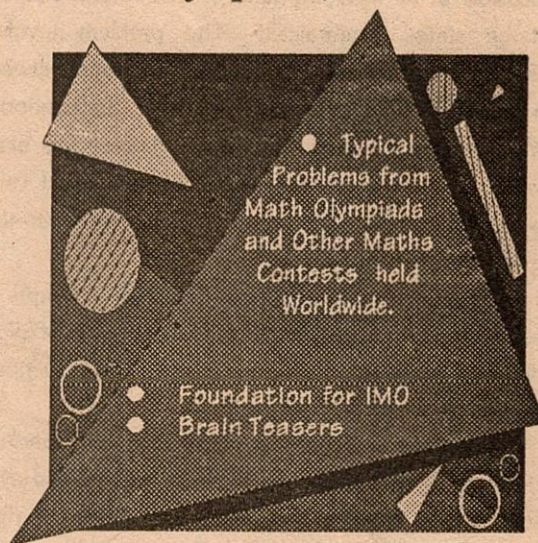
— GEOTHE

".....Mathematical proofs, like diamonds, are hard as well as clear, and will be touched with nothing but strict reasoning."

— JOHN LOCKE

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Multiple Choice Questions

- $\sin^2 \theta = \frac{2xy}{x^2 + y^2}$ is true if and only if
(a) $x = y$ (b) $x > y$
(c) $x < y$ (d) $x = 0$.
- If $F(x) = \tan \left\{ 2 \tan^{-1} \left(\frac{1}{5} \right) - \frac{\pi}{5} \right\}$ and $g \left(\frac{-7}{17} \right) = 1$ then $(g \circ F) x$ is equal to
(a) 2 (b) 1
(c) $\pi/4$ (d) none of these.
- If the alternate angles of a regular pentagon are joined forming another regular pentagon; then the ratio of the areas of the two pentagons is
(a) $2 : 7 - 3\sqrt{5}$ (b) $\sqrt{2} : 7 - 5\sqrt{3}$
(c) $7 - 3\sqrt{5} : 2$ (d) $7 - 5\sqrt{3} : \sqrt{2}$.
- If $\sin \left(\log_e^i \right) + \cos \left(\log_e^i \right) = \cos \theta$ then the value of θ is
(a) 90° (b) 180°
(c) 45° (d) 0° .
- If θ, ϕ and x are the angles of a triangle and $e^{i\theta}, e^{ix}, e^{i\phi}$ are in A.P., then the triangle is
(a) Isosceles (b) Right angled
(c) Equilateral (d) None.
- If $y = \cot^{-1} \left\{ \frac{\log(ex^2)}{\log \left(\frac{e}{x^2} \right)} \right\} + \tan^{-1} \left(\frac{3 + 2 \log x}{1 - 6 \log x} \right)$ then the value of $\frac{d^2y}{dx^2}$ at $x = 1$ is
(a) -1 (b) 0
(c) 1 (d) e.
- If the equations $ax^2 + 2bx + c = 0$ and $x^2 + 2P^2x + 1 = 0$ have a common root, where, a, b, c are in AP and $P^2 \neq 1$, then the second root of the second equation is
(a) $x = -\frac{c}{a}$ (b) $x = \frac{c}{a}$
(c) $x = -\frac{a}{c}$ (d) $x = \frac{a}{c}$.
- The number of tangents which can be drawn from the point $(-1, 2)$ to the circle $x^2 + y^2 + 2x - 4y + 4 = 0$
(a) 1 (b) 2 (c) 0 (d) 3
- If $y = e^{-\sin x}$ then the maximum value of y is
(a) 0.366 (b) 2.73
(c) $\log \sin x$ (d) none of these.
- The value of θ if $[e^{i\theta}]^2$ is purely imaginary is
(a) $\theta = 2n\pi \pm \frac{\pi}{4}$ (b) $\theta = n\pi \pm \frac{\pi}{4}$
(c) $\theta = 2n\pi$ (d) $\theta = n\pi$.
- If the products of the roots of the equation $(\log_2 x)^2 - 3K(\log_2 x) + 2e^{i\theta} - 1 = 0$ is $e^{i\theta}$ then the roots are real. The general value of θ is
(a) $\theta = 2n\pi$ (b) $\theta = 2n\pi \pm \frac{\pi}{2}$
(c) $\theta = n\pi + (-1)^n \frac{\pi}{2}$ (d) $\theta = n\pi$.
- $\int x^{-3} \times 5^{1/x^2} dx = K \times 5^{1/x^2}$ then K is
(a) $-\frac{1}{2 \log 5}$ (b) $-2 \log 5$
(c) $\frac{2}{\log 5}$ (d) $-\frac{2}{\log 5}$.
- The area bounded by the parabola $y = x^2$ and the lines $x + y + 2 = 0$ and $x = 2$ is
(a) $\frac{8}{3}$ (b) $\frac{22}{3}$
(c) $\frac{28}{3}$ (d) $\frac{40}{3}$.
- If $\sin \theta + \operatorname{cosec} \theta = 2$ then the value of

$\sin^n \theta + \operatorname{cosec}^n \theta$ is

- (a) 2 (b) $2n$ (c) n (d) 0

15. The value of $\sum_{n=1}^{200} i^n$ where $i = \sqrt{-1}$

- (a) 50 (b) -50 (c) 0 (d) none.

16. In a triangle ABC if $A = \frac{\pi}{7}$, $B = 2A$, $C = 2B$

then the value of $a^2 + b^2 + c^2$ is

- (a) $3R^2$ (b) $5R^2$ (c) $7R^2$ (d) $9R^2$

17. The derivative of X^{X^X} with respect to $(X^X)^X$ is

(a) $X^{(X^X - x^2 + x - 2)} \left\{ \frac{1 + (\log ex)(\log x)}{\log(ex^2)} \right\}$

(b) 0

(c) $X^{(X^X + x^2 - x + 2)} \left\{ \frac{1 + (\log ex)(\log x)}{\log(ex^2)} \right\}$

(d) none.

18. Three vectors of magnitudes a , $2a$, $3a$ meet in a point and their directions are along the diagonals of three adjacent faces of a cube, then their resultant is

- (a) $6a$ (b) $7a$ (c) $5a$ (d) $3a$

19. The value of $\lim_{x \rightarrow \infty} \frac{e^x - (1+x)}{x^2}$ is

- (a) 0 (b) e (c) 2 (d) $1/2$.

20. If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 3! & \sin 3 \frac{\pi}{2} & \cos 3 \frac{\pi}{2} \\ K & K^2 & K^3 \end{vmatrix}$

then the value of $\frac{d^3}{dx^3}[f(x)]$ at $x = 2\pi$ corresponds to

- (a) -1 (b) 0
(c) 1 (d) independent of K

PART B

1. Solve the equations in θ and ϕ

$2^{\sin \theta + 1} \cos \phi = 6^{\cos \phi}$

$3^{\sin \theta} = 3.2^{\cos \phi + 1}$

2. Evaluate

$\int_0^\pi \log(1 - 2a \cos \theta + a^2) \cos n \theta d\theta$ where $a^2 < 1$.

3. The area bounded by the line $y = x$ the curve $y = f(x)$ above it, and the lines $x = 1$ and $x = t$, is equal to

$\left(\sqrt{t^2 + 1 + t + t} \right)^{3/2}$ for all $t > 1$. Find $f(x)$.

4. If $x > 0$ show that $x - \frac{x^3}{2} < \log(1+x) <$

$x - \frac{x^2}{2(1+x)}$

5. Prove that

$\int \frac{2e^t dt}{(e^{3t} - 6e^{2t} + 11e^t - 6)} = \log \left\{ \frac{(e^t - 1)(e^t - 3)^2}{(e^t - 2)} \right\} + C$

6. A monkey is climbing a mast of a ship at $2\sqrt{2}$ feet per second. If the ship is moving at 5 feet per second towards south and the current is taking it towards east at 4 feet per second towards east; find the velocity of the monkey in space.

7. Solve for θ if $e^{ie^{i\theta}} = 0$.

8. For all x in $[0, 1]$, let $f''(x)$ exist and satisfy

If $|f''(x)| < 1$ if $f(0) = f(1)$, show that

If $|f'(x)| < 1$ for all x in $(0, 1)$.

ANSWERS

PART A

1. a, b, c 2. (b) 3. (c) 4. (b) 5. (c)
6. (b) 7. (b) 8. (c) 9. (a) 10. (b)
11. (a), (c) 12. (a) 13. (d) 14. (a) 15. (c)
16. (d) 17. (a) 18. (c) 19. (d) 20. (b)

PART B

1. $\phi = \cos^{-1} \left[\pm \frac{\log 2}{\log 3 - \log 2} \right]$

$\theta = \sin^{-1} \left[\pm \frac{\log 2}{\log 3 - \log 2} \right]$ 2. $-\frac{a^n \pi}{n}$

3. $f(x) = x + \frac{3}{4\sqrt{x^2 + 1}} \left(\sqrt{x^2 + 1} + x + x \right)^{1/2}$
 $\left(\sqrt{x^2 + 1} + x + 2\sqrt{x^2 + 1} \right)$

6. velocity = 7 feet per second.

7. $\theta = \frac{\pi}{2}$, $\theta = \cos^{-1} \left[\frac{\pi}{2} - \cos^{-1} \left(\frac{\pi}{2} \right) \right]$

100 Questions on

Limits, continuity & Differentiability

OBJECTIVE/SHORT ANSWER QUESTIONS

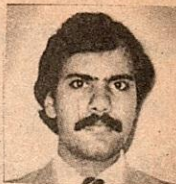
Evaluate the limit in all questions unless otherwise specified

1. $\lim_{x \rightarrow 0} \frac{\sin(a+x) - \sin(a-x)}{\tan(a+x) - \tan(a-x)}$
2. $\lim_{x \rightarrow 1} \left(\frac{1+x}{2+x} \right)^{\frac{(1-x)}{(1-x)}}$
3. $\lim_{x \rightarrow 1} \frac{(7+x^3)^{1/3} - \sqrt{3+x^2}}{x-1}$
4. $\lim_{x \rightarrow 0} (1 + 3\tan^2 x)^{\cot^2 x}$
5. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$
6. $\lim_{x \rightarrow 0} \frac{(1+x^2)^{1/3} - (1-2x)^{1/4}}{x+x^2}$
7. $\lim_{x \rightarrow 0} (\cot x - (1/x))$
8. $\lim_{x \rightarrow 0} [\tan((\pi/4) + x)]^{1/x}$
9. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\log(1+x)}$
10. $\lim_{x \rightarrow \pi/2} (1 + \cos x)^{\sec x}$
11. Find $\lim_{x \rightarrow 0} \frac{\sqrt{1/2(1 - \cos 2x)}}{x}$ if it exists
12. $\lim_{x \rightarrow a} \frac{(a^2 - x^2)^{1/4} - (a-x)^{3/2}}{(a^3 - x^3)^{1/4} - (a-x)^{3/4}}$
13. $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + 4^3 \dots n^3}{n^2 + 2n^2 + 3n^2 + 4n^2 \dots n^3}$
14. $\lim_{x \rightarrow e} \frac{\log x - 1}{x - e}$
15. $\lim_{x \rightarrow 0} \left(\frac{a+x}{a-x} \right)^{1/x}$
16. $\lim_{x \rightarrow \infty} \left[\frac{1}{1.2} + \frac{1}{1.3} + \dots + \frac{1}{(n-1)n} \right]$
17. $\lim_{x \rightarrow 1} \frac{1-x+\log x}{1-\sqrt{2x-x^2}}$
18. $\lim_{x \rightarrow 0} x^x$
19. If $\lim_{x \rightarrow \infty} \left(\frac{x+9}{x-c} \right) = 4$; find the value of c .

20. $\lim_{x \rightarrow \infty} \sqrt{(x+a)(x+b)} - x$
21. $\lim_{x \rightarrow \infty} \frac{1 + (1/2) + (1/4) \dots (1/2^n)}{1 + (1/3) + (1/9) \dots (1/3^n)}$
22. $\lim_{x \rightarrow \infty} \left(\frac{x}{2+x} \right)^{2x}$
23. $\lim_{x \rightarrow 0} \frac{e^x - (1+x)}{x^2}$
24. Given that : $f(x) = \frac{1}{x^2}$ $g(x) = \frac{1}{x^4}$,
find the value of $\lim_{x \rightarrow 0} [f(x) - g(x)]$
25. Examine the following function for continuity at $x = 0$
$$f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$$
26. $\lim_{x \rightarrow 0} \frac{\sin(a+2x) - 2\sin(a+x) + \sin a}{x^2}$
27. Examine continuity at $x = 1$
$$f(x) = \frac{x-1}{1+e^{1/(x-1)}}; \quad x \neq 1 \quad f(1) = 0$$
28. $\lim_{x \rightarrow 0} \frac{\log(1+x^2+x^4)}{3x^2(1-2x)}$
29. $\lim_{x \rightarrow 0} \frac{\sin(1+x) - \sin(1-x)}{x}$
30. $\lim_{x \rightarrow a^+} \frac{[x]}{x}; \quad a > 0, a \in \mathbb{I}$
31. $\lim_{x \rightarrow 0} \frac{\log(1+kx^2)}{1-\cos x}$
32. Examine continuity at $x = 0$
$$f(x) = \begin{cases} \frac{e^{x^2}}{e^{1/x^2} - 1}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$
33. $\lim_{x \rightarrow \infty} (1 + (1/x))^{(x+1)/x}$

How to Study

for Engineering Entrance Exams



Dear friend,

My name is Raj Bapna. In this letter, I have something very important to say that can help you greatly to get success in your exams and competitions. If you have 10 minutes, I request you to read this page about how to study, how to use more mind power, how to improve your memory and much much more.

Yogis have always known it and scientists have also discovered it now—that most people use only 10% of their mind power.

How Will You Benefit

Before you read this page fully, I want to make it clear that my course cannot give success by magic. But with my course, you can be more sure of success because you become better than 99% of students in the following **9 Critical Success Factors**:

1. Good increase in your memory and concentration
2. Your effectiveness to read and learn will increase greatly
3. Your ability to study longer without getting tired (body or mind) or feeling sleepy will increase
4. You will experience that you are capable of achieving much more success than you currently do (even if you are already very good)
5. Small to moderate improvement in your intelligence
6. Set realistically high aims/goals and take you step by step on the road to achieve success
7. Improve writing, spelling, interview skills
8. Learn exam secrets to get more marks for what you have studied
9. Avoid big mistakes that can result in failure.

Suppose you improve only 5% in each, then total improvement is $5 \times 9 = 45\%$. I know you will improve 100% just in reading speed. So, total improvement will be great for your success.

Simple, Practical, Effective

My techniques are effective. They do not make you tired. And you can learn them fast. I teach no theory. Only the techniques that have proved effective for myself and other students.

You may find it difficult to fully understand the power and benefits of my course just by reading this page now. But, those who join my course will benefit greatly and avoid mistakes that can cause failure for others.

The newspaper **Times of India**, says that from my course you learn "Simple, effective, practical techniques to improve overall intelligence and mind power. Even average student can easily understand."

New All India Memory Record

One of our students R Caudally has recently set All India Memory Record. In interviews to many newspapers he said "The secret of my newly developed memory are postal courses **Mind Power Music** and **Mind Power Study Techniques** from the Mind Power Research Institute, Udaipur." Before joining our courses, he was an average student and scored only 52.3% in High School Exam.

Are you thinking now "If my course can help someone to set a new memory record, can it not help me to get more success in your exam?"

Improve Your Memory Quickly

Of many easy techniques, I explain two: **ONE**. The brain has two memory stores: short-term and long-term. Research shows that without revision, after 24 hours we remember only 18%. After 1 month only 5%. It clearly shows that we must revise. But, most students do not revise systematically, so much of their hard work is wasted. I teach you the powerful techniques "Systematic Revision" and "Daily Routine" so that you can revise and remember more in less time.

TWO. Scientific research has proved that for better memory, we should take rest and not study continuously for hours. You will learn my technique "Rest Routine" to get maximum benefit from the rest. This technique relaxes you, changes your brain waves, and puts you in a "learning state".

Read Faster to Revise Faster

Everyone can learn to read and understand 300, or 500 or more words per minute. But, many of us read only about 100 words per minute. My "Finger Technique" will double your reading speed in 30 minutes. Your read slow if:

- If you read aloud or move lips
- If you do not read aloud but hear the sounds in your mind when you read
- If you read one word at a glance rather than reading many words at a glance
- If, without your knowledge, you read some words again and again.

My course will help you to overcome these habits. The best use of reading faster is not to study new chapters for the first time, but to revise again and again quickly so that you can remember more in less time. The "Finger Technique" helped me to increase my reading speed from 72 to 1037 words per minute. Here is what two experts say about this technique:

"I am very happy to inform you that my son Ravi Anand increased his reading speed from 228 to surprisingly high 1818 words per minute. Thank you for your course."

—Dr M L Singh, MBBS, MS, Eye Surgeon, Bihar
"Unbelievably, I improved my reading speed from 75 to 200 words per minute. My son improved his memory. He also improved his reading speed from 45 to 100."

—Prof M Bhatnagar, PhD, Formerly in USA

I Was Not Always Successful

I want you to know that I was not always highly successful as a student. You can call it luck or chance that I happened to discover a few techniques to study for success. These techniques changed my life and my marks improved in three years from 73.0% to 78.0%, 83.5%, 87.7%. Similarly, I did not get NTS scholarship in class 10 because I made a simple, stupid mistake. Then in class 11, I did not make the mistake and I got success in NTS.

Do you realize that if just a few techniques improved my success so much, what my complete course can do for your success?

What is Unique About It

My course combines 5000 year old Indian techniques with the latest scientific discoveries in brain research, nutrition, psychology, and music in America and other countries.

In USA, just before returning to India, I spent 1300 dollars (about Rs 42,000) to join two courses to learn 3 more mind power techniques. You will learn them in my course. My personal library has books and courses worth Rs 1,17,210. I have read, experimented, researched with all their techniques and included only the best ones into my course. These techniques are in addition to my own developed techniques in the course. This course is protected by the Copyright Law, and no body can copy my material.

Used by Lakhs World-Wide

Lakhs of people from every corner of India and from many parts of the world are benefiting everyday from my course. Consider just this simple fact: If a course from India is used even abroad, the course must be really good.

Do you understand fully that you can decide to order this course now to help you to get success and also to fulfil your parent's hopes and dreams?

Music for Success

It has powerful effects on your mind/brain. So, it is not for people with epilepsy, and anyone undergoing psychiatric or electro-therapy.

It is based on scientific research into how the mind works and how to program and control it for our own success. It has sounds from instruments and nature (river, birds). For details on how such relaxing music helps to learn faster, please read USA best-seller books "Superlearning" and "MegaBrain".

The Hidden-Messages™ in music bypass your

BIO-DATA

- B.E., BITS Pilani, M.Tech, IIT Kharagpur, NTSE scholar. Rank 5 Raj School Board.
- World-famous author. I published 3 computer books in USA. One is best selling (*MS-DOS Masters*).
- Increased my reading speed from 72 to 1037 words per minute. Was a member of *Society for Accelerated Learning & Teaching, USA*.
- My first job as an engineer paid only Rs 1000 per MONTH. Just 7 years later, I earned 50 dollars per HOUR in USA as computer expert.
- At the peak of success, I returned to India to do something in our own country. Now, I spend my full time as a scientist to do mind power research.
- I also learnt French, Sanskrit, Karate, Breaking wooden board by hand, many Meditations, etc.

conscious mind and go to your subconscious mind, and change your behaviour. Here is what people say:

"I have already purchased a course of Mind Power Music. Please send me 6 more for the use of my staff. Thank you."

—Rector (Principal), Holy Rock School, Burdwan, W.B.

"Very good. It relaxes my body and mind. It reduces the tension of my studies."

—Dr Anju Banthiya, MBBS, Bhopal

Money-Back GUARANTEE

Order my course (code 805 or 712) and if you are not 100% satisfied, tear it into 2 pieces and return in 31 days, and I will send MO for your money (less Rs 30 for processing charge). No questions will be asked. If you order this month, I will also send a poster of Bapna's Optical Illusion™ Technique for Concentration. Keep it as free gift even if you return the course.

Time Does Not Wait for Anyone

It is now upto you. You can turn this page as if you did not even read it and miss this opportunity for more success. Or, you can join this course today. Will the coming weeks and months make you a much better student by joining this course? Or, will you remain like many others and struggle for success?

You decide.

Discount Prices

Course Name	Course Code	Price + Postage
Mind Power		
Mind Power Study Techniques course	805	145+15
Memory and Concentration cassette (FREE: get book Mind Power Music)	110	66+10
Medical / Engineering Subjects		
Memory Maps for Physics	510	225+15
Memory Course for Chemistry (FREE: get book Advanced Numericals and Questions and Answers in Chemistry)	520	225+15
Memory Course for Maths	550	225+15
Special Offers (Medical/Engineering)		
Engineering 3 courses (510, 520, 550)	922	675+15
All of the 5 courses above (2 mind power and 3 engineering)	922+712	885+15

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$$34. \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{\sin x}{x}}$$

$$35. \text{Find } \lim_{x \rightarrow 0} e^{1/x} \text{ if it exists}$$

$$36. \text{Examine continuity at } x = 0$$

$$f(x) = \begin{cases} \frac{e^{1/x}}{1 + e^{1/x}} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$37. \lim_{x \rightarrow \infty} \{\cos(x/2) \cdot \cos(x/4) \cdot \cos(x/8) \dots \cos(x/2^n)\}$$

$$38. \text{Examine continuity at } x = 0$$

$$f(x) = x \sin(1/x); x \neq 0$$

$$f(0) = 0$$

$$39. \lim_{x \rightarrow a} \frac{a^x - x^a}{x^x - a^a}$$

$$40. \lim_{x \rightarrow 0} (\sin x + \cos x)^{1/x}$$

$$41. \lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x - 1}{2x^2 - 3x - 2} \right)^{\frac{2x+1}{x-1}}$$

$$42. \text{Examine continuity at } x = a \text{ (} a > 0 \text{)}$$

$$f(x) = \begin{cases} [x]; & x \neq a \\ 0 & x = a \end{cases}$$

$$43. \lim_{x \rightarrow \infty} x \left(\tan^{-1} \frac{x+1}{x+2} - \frac{\pi}{4} \right)$$

$$44. \text{Find: } \lim_{x \rightarrow \infty} a^x \sin(b/a^x) \\ \text{when (i) } 0 < a < 1 \quad \text{(ii) } a > 1$$

$$45. \lim_{h \rightarrow 0} \frac{f(x+h) - |x|}{h}$$

$$46. \lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{1/x}$$

$$47. \lim_{n \rightarrow \infty} (3^n + 5^n)^{1/n}$$

$$48. \lim_{x \rightarrow \infty} \frac{3x^3 + 2}{\sqrt{x^4 - 2}}$$

$$49. \lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2} \right)^x$$

$$50. \lim_{x \rightarrow 1} \frac{x^x - x}{1 - x + \log x}$$

$$51. \lim_{x \rightarrow a} \frac{e^{mx} - e^{ma}}{x - a}$$

$$52. \lim_{x \rightarrow \infty} x^n e^{-x/n}; n > 0, n \in \mathbb{I}$$

$$53. \text{Give all the points of continuity of}$$

$$f(x) = \{x\} = x - [x]$$

$$54. \lim_{x \rightarrow 0} \frac{\tan(a+2x) - 2\tan(a+x) + \tan a}{x^2}$$

$$55. \text{Give all the points of discontinuity of: } f(x) = [x]$$

$$56. \text{Examine continuity at } x = 0$$

$$f(x) = \frac{1 + 2^{1/x}}{2 - 2^{1/x}}; x \neq 0 \quad f(0) = -1$$

$$57. \text{For what values of } x \text{ is the following continuous?}$$

$$f(x) = |x| + |x - 1|$$

$$58. \lim_{x \rightarrow \infty} 6x^{12} - 8x^7 + 9x + 1$$

$$59. \lim_{x \rightarrow 1} \operatorname{cosec} \pi x \log x$$

$$60. \lim_{x \rightarrow \infty} \frac{3x - 4}{\sqrt[3]{x^2 - 1}}$$

$$61. \lim_{x \rightarrow a^+} \frac{1}{x - a} \int_a^x f(t) dt$$

$$62. \lim_{x \rightarrow \infty} \frac{2^x}{2^{10}}$$

$$63. \lim_{x \rightarrow 2a} \frac{\sqrt{x} - \sqrt{2a} + \sqrt{x - 2a}}{\sqrt{x^2 - 4a^2}}$$

$$64. \lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4}$$

$$65. \text{Examine continuity at } x = 0$$

$$f(x) = \begin{cases} \frac{2x^2 + |x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$66. \lim_{x \rightarrow \infty} 3x^7 - 5x^5 - 7x^3 - 9x$$

$$67. \lim_{x \rightarrow \infty} \left\{ \left(\frac{n+1}{n} \right)^n - \frac{n+1}{n} \right\}^n$$

$$68. \text{Let } f: \mathbb{R} \rightarrow \mathbb{R} \text{ be a differentiable function and}$$

$$f(1) = 4 \text{ Then, find } \lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt$$

$$69. \text{Given } f(x) = [\tan^2 x] \text{ Then}$$

$$(i) \lim_{x \rightarrow 0} f(x) \text{ does not exist}$$

$$(ii) f(x) \text{ is continuous at } x = 0$$

$$(iii) f(x) \text{ is not differentiable at } x = 0$$

$$(iv) f(0) = 1$$

$$70. \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x}$$

$$71. \text{Which of the following functions are continuous on } (0, \pi)$$

$$(a) \tan x \quad (b) \int_0^x t \sin(1/t) dt$$

$$(c) f(x) = \begin{cases} 1 & 0 < x < (3\pi/4) \\ 2 \sin(2x/9) & (3\pi/4) < x < \pi \end{cases}$$

$$(d) f(x) = \begin{cases} \sin x & 0 < x \leq \pi/2 \\ \pi/2 \sin(\pi + x) & \pi/2 < x < \pi \end{cases}$$

$$72. \text{ Find } \lim_{x \rightarrow 1} \frac{d/dx |x^2 - 1|}{|x - 1|}$$

if it exists

$$73. \text{ If } \lim_{x \rightarrow 0} \frac{a^3 + 8e^{2x}}{1 - b^3e^{1/x}} = 2,$$

then $a = \dots\dots\dots$ and $b = \dots\dots\dots$

$$74. \text{ Given } f(a) = 2; f'(a) = 1 \\ g(a) = -1; g'(a) = 2$$

$$\text{Then find } \lim_{x \rightarrow a} \frac{g(x)f(a) - f(x)g(a)}{x - a}$$

$$75. \text{ If } f(x) = e^{-x^n} \quad x \neq 0, \quad f(0) = 0$$

Then f will be differentiable at $x = 0$ for what n ?

$$76. \lim_{x \rightarrow 1} (1 - x) \tan \frac{\pi x}{2}$$

$$77. \lim_{x \rightarrow 0} [\cot(\frac{\pi}{4} - x)]^{\sin(1/x)}$$

$$78. \lim_{x \rightarrow 0} \frac{\sqrt{a^2 + ax + x^2} - \sqrt{a^2 - ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$

$$79. \text{ Let } f(x) = \begin{cases} 3x^2 - 1 & x < 0 \\ cx + d & 0 \leq x \leq 1 \\ \sqrt{x} + 8 & x > 1 \end{cases}$$

Determine c & d so that f is continuous everywhere

$$80. \lim_{x \rightarrow 0} (1 + \tan^2 \sqrt{x})^{\frac{1}{\sqrt{x}}}$$

$$81. \lim_{x \rightarrow \infty} (\cos \sqrt{x} - \cos \sqrt{x+1})$$

$$82. \lim_{x \rightarrow 1} \frac{1 - x + \log x}{1 + \cos \pi x}$$

SUBJECTIVE/LONG ANSWER QUESTIONS

$$83. \text{ If } f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

show that the function is continuous and differentiable for $x = 0$

$$84. \lim_{x \rightarrow 0} \frac{\cos(\sin x) - \cos x}{x^4}$$

$$85. \text{ If } |x| < 1; \text{ prove that } \lim_{n \rightarrow \infty} [(1+x)(1+x^2)(1+x^4) \dots (1+x^{2^n})] = \frac{1}{1-x}$$

$$86. \text{ Prove that there exists a solution of } \cos x = x^n \text{ in } [0, \pi/2], \quad n \in \mathbb{I}$$

$$87. \text{ Examine derivability at } x = 0$$

$$f(x) = \begin{cases} x^m \sin(1/x), & x \neq 0, \quad m > 0 \\ 0, & x = 0 \end{cases}$$

Determine m when $f(x)$ is continuous at $x = 0$

$$88. \text{ Discuss limit, continuity and differentiability}$$

$$f(x) = \begin{cases} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$89. \text{ Draw the graph of the following function and discuss its continuity and differentiability at } x = 1$$

$$f(x) = \begin{cases} 3^x & -1 \leq x \leq 1 \\ 4 - x & 1 < x < 4 \end{cases}$$

$$90. \text{ Evaluate the limit and test continuity at } x = 0;$$

$$f(x) = \begin{cases} x \cdot \left(\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$91. \text{ A function is defined as :}$$

$$f(x) = \begin{cases} x^3, & x^2 < 1 \\ x, & x^2 \geq 1 \end{cases}$$

Draw the graph of the function and discuss continuity and differentiability at $x = 1$

$$92. \text{ Show that the function}$$

$$f(x) = \begin{cases} x [1 + (1/3) \sin(\log x^2)] & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is continuous but has no derivative at $x = 0$

$$93. \text{ If } f(x) = \sqrt{|x - 1|} \text{ and } g(x) = \sin x \\ \text{Find } (f \circ g)(x) \text{ and } (g \circ f)(x) \text{ and discuss the differentiability of } (g \circ f)(x) \text{ at } x = 1$$

$$94. \text{ If } f(x) = -1 + |x - 1|, \quad -1 \leq x \leq 3$$

$$g(x) = 2 - |x + 1|, \quad -2 \leq x \leq 2$$

Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$, Draw their graphs and discuss continuity of $(f \circ g)(x)$ at $x = -1$ and differentiability of $(g \circ f)(x)$ at $x = 1$

$$95. \text{ Show that } f'(x) \text{ has a finite limit at } x = 0$$

$$f(x) = \frac{\sin x^2}{x}, \quad f(0) = 0$$

$$96. \text{ Let } f(x) \text{ be a continuous function such that } f(x + y) = f(x) + f(y) \quad \forall x, y \in \mathbb{R}. \text{ Prove that } f(x) = cx \text{ given that } c \text{ is a constant } f(1) = c, \quad x \in \mathbb{I}, \quad x > 0$$

97. If the function $f(x) = \begin{cases} \frac{\sqrt{x} - \sqrt{a}}{\sqrt[3]{x} - \sqrt[3]{a}} & x \neq a \\ k & x = a \end{cases}$

is continuous at $x = a$, what is the value of k ?

98. Let $g(x)$ be a polynomial of degree one and $f(x)$ be defined by :

$$f(x) = \begin{cases} g(x); & x \leq 0 \\ \left(\frac{1+x}{2+x}\right)^{1/x}; & x > 0 \end{cases}$$

Find the continuous function $f(x)$ satisfying $f(1) = f(-1)$

99. Let $f(x)$ be an even function for all x . If $f(0)$ exists, find its value.

100. Let $f(x) = \begin{cases} \{1 + |\sin x|\}^{a/|\sin x|} & -\pi/6 \leq x < 0 \\ b & x = 0 \\ e^{\tan 2x/\tan 3x} & 0 < x \leq \pi/6 \end{cases}$

Determine a and b such that $f(x)$ is continuous at $x = 0$.

Solutions/Answers/Hints

Objective Problems

1. Ans. $\cos^3 a$

(Hint : given limit $= \lim_{x \rightarrow 0} \frac{\sin(a+x) - \sin(a-x)}{\frac{\sin(a+x)}{\cos(a+x)} - \frac{\sin(a-x)}{\cos(a-x)}}$)

$$= \lim_{x \rightarrow 0} \frac{2\cos a \sin x \cos(a+x)\cos(a-x)}{[\sin(a+x)\cos(a-x) - \sin(a-x)\cos(a+x)]}$$

(Den. $= \sin 2x = 2\sin x \cos x$)

2. Ans. $\frac{\sqrt{2}}{3}$ (Hint : $\frac{1 - \sqrt{x}}{1 - x} = \frac{1}{1 + \sqrt{x}}$)

3. Ans. $-1/4$

(Hint : Use binomial theorem or L-Hospital's rule)

4. Ans. e^3

(Hint : Limit $= \lim_{x \rightarrow 0} (1 + 3\tan^2 x)^{\frac{1}{\tan^2 x}}$)

Put $\tan^2 x = y$; $x \rightarrow 0 \Rightarrow y \rightarrow 0$)

5. Ans. $1/2$

(Hint : Apply L-Hospital rule as it is $0/0$ form. Then,

$$= \lim_{x \rightarrow 0} \frac{1 + x^2 - \sqrt{1 - x^2}}{3x^2\sqrt{1 - x^2}(1 + x^2)}$$

Rationalise numerator and eliminate x^2)

6. Ans. $1/2$

(Hint : Apply binomial theorem or use L - H rule)

7. Ans. 0

(Hint : Limit $= \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{\sin x \cdot x}$)

$$= \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{(\sin x/x) \cdot x^2} = \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^2}$$

$0/0$ form. Apply LH rule)

8. Ans. e^2

(Hint : Given $\lim_{x \rightarrow 0} \left[\frac{1 + \tan x}{1 - \tan x} \right]^{1/\tan x}$)

$$e/e^{-1} = e^2$$

9. Ans. 2

(Hint : $0/0$ form. Apply LH rule once)

10. Ans. e^3

11. Ans. Limit does not exist

(Hint : given $\lim_{x \rightarrow 0} \frac{|\sin x|}{x}$)

$$L.H.L \neq R.H.L$$

12. Ans : $\frac{\sqrt{2a}}{\sqrt{3a+1}}$

(Hint : eliminate $\sqrt{a-x}$ from denominator and numerator)

13. Ans. $1/2$

(Hint : Sum the series in the numerator, series in denominator $= n^2(1 + 2 + \dots + n) = n^3(n+1)/2$. Now divide above and below by n^4 .)

14. Ans : $1/e$

(Hint : $0/0$ form. Apply LH rule once)

15. Ans. $e^{2/a}$

(Hint : Given $\lim_{x \rightarrow 0} [(1 + (x/a))^{1/x}] [(1 - (x/a))^{-1/x}]$)

$$= e^{1/a} \cdot e^{1/a} = e^{2/a}$$

16. Ans. 1

(Hint : Break the series into partial fractions.

$$\text{Given} = \lim_{n \rightarrow \infty} \left[\sum_{m=2}^n \left(\frac{1}{m-1} - \frac{1}{m} \right) \right]$$

$$= \lim_{n \rightarrow \infty} [(1 + (1/2) + (1/3) \dots (1/n)) - ((1/2) + (1/3) \dots 1/(n-1) + 1/n)]$$

$$= \lim_{n \rightarrow \infty} (1 - (1/n)) = 1$$

17. Ans. -1

(Hint : 0/0 form. Apply LH rule)

18. Ans. 1

(Hint : Note that 0^0 is an indeterminate form. Given

limit = $\lim_{x \rightarrow 0} \exp(\log x^x)$

[where, $\exp(x) = e^x$, also note that $e^{\log x} = x$]

$$= \lim_{x \rightarrow 0} \exp\left(\frac{\log x}{1/x}\right)$$

$-\infty/\infty$ form. Apply L.H. rule

$$= \lim_{x \rightarrow 0} \exp((1/x) \cdot (-x^2)) = \exp(0) = 1$$

19. Ans. $c = \log 2$

(Hint : Put $1/x = t$; $x \rightarrow \infty \Rightarrow t \rightarrow 0$

$$\text{then } \lim_{t \rightarrow 0} \frac{(1/t) + 9^{1/t}}{(1/t) - c} = \lim_{t \rightarrow 0} (1 + ct)^{1/t} \cdot (1 - ct)^{-1/t}$$

$$= e^c \cdot e^c = e^{2c} \quad e^{2c} = 4 \Rightarrow c = \log 2$$

20. Ans. $\frac{a+b}{2}$

(Hint : Given limit after rationalizing

$$= \lim_{x \rightarrow \infty} \frac{n(a+b) + ab}{\sqrt{(x+a)(x+b)} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{a+b + (ab/x)}{\sqrt{(1+(a/x))(1+(b/x))} + 1} = \frac{a+b}{2}$$

21. Ans. 3/4

(Hint: Sum the G.P.s in the numerator and denominator)

22. Ans. e^{-4}

(Hint : Given $\lim_{x \rightarrow \infty} \left(\frac{2+x}{x}\right)^{2x}$

$$= \lim_{x \rightarrow \infty} [(1 + (2/x))^x]^{-2} = [e^2]^{-2} = e^{-4}$$

23. Ans. 1/2

(Hint : 0/0 form. Apply L.H. rule)

24. Ans. $-\infty$

(Hint : Given $\lim_{x \rightarrow 0} ((1/x^2) - (1/x^4))$

$$= \lim_{x \rightarrow 0} (x^2 - 1)/x^2 = (0 - 1)/0 = -\infty$$

25. Ans. Not continuous

(Hint : Limit = 0; $f(0) = 1$)

26. Ans. $-\sin x$

(Hint : 0/0 form. Apply L.H. rule)

27. Ans : continuous

(Hint : Limit = $0/\infty = 0 = f(1)$)

28. Ans. 1/3

(Hint : 0/0 form. Apply L.H. rule directly)

29. Ans : $2\cos 1$

(Hint : Given $\lim_{x \rightarrow 0} \frac{2 \sin x \cos 1}{x}$)

30. Ans. 1

(Hint : If $a > 0$; $[a^x] = a$)

31. Ans : -1/2

(Hint : 0/0 form. Apply L.H. rule)

32. Not continuous

(Hint : $\lim_{x \rightarrow 0} \frac{e^{x^2}}{e^{1/x^2} - 1} = 0/\infty - 1 = 0 \neq f(0)$)

33. Ans = 1

(Hint : Given = $\lim_{x \rightarrow \infty} (1 + (1/x))^{1 + (1/x)}$

Put $1 + (1/x) = t$; $x \rightarrow \infty \Rightarrow t \rightarrow 1$)

34. Ans. 1/e

(Hint : Put $\sin x/x = t$; $x \rightarrow 0 \Rightarrow t \rightarrow 1$

$$\text{Now } \frac{\sin x}{x - \sin x} = \frac{t}{1-t}$$

$$\therefore \text{Given limit} = \lim_{t \rightarrow 1} (t)^{1/(1-t)} = \lim_{t \rightarrow 1} \exp\left(\frac{1}{1-t} \log t\right)$$

$$= \lim_{t \rightarrow 1} \exp\left(\frac{t \log t}{1-t}\right)$$

0/0 form. Apply L.H. rule

$$= \lim_{t \rightarrow 1} \exp\left(\frac{1 + \log t}{-1}\right) = e^{-1}$$

35. Ans. Limit does not exist

(Hint : R.H.L = $\infty \neq \infty$ = L.H.L)

36. Ans: Not continuous

(Hint : $\lim_{x \rightarrow 0} f(x)$

$$= \lim_{x \rightarrow 0} \frac{e^{-2/x}}{e^{-1/x} + 1} = 0 \neq f(0)$$

37. Ans: $\sin x/x$

(Hint : Given limit $= \lim_{n \rightarrow \infty} \left(\frac{\sin x}{2^n \sin(x/2^n)} \right)$

(By trigo. result) $= \lim_{n \rightarrow \infty} \left(\frac{2^n \sin x}{\sin(x/2^n)} \right)$

0/0 form. Apply L.H. rule taking n as variable)

38. Ans : continuous

(Hint : $\lim_{x \rightarrow 0} f(x) = 0 \times$ (some value between -1 and 1)
 $= 0 = f(0)$)

39. Ans : $\frac{(\log a) - 1}{(\log a) + 1}$

(Hint : 0/0 form. Apply L.H. rule)

40. Ans. e

(Hint : Limit $= \lim_{x \rightarrow 0} (1 + \sin 2x)^{1/2x}$

(Squaring and taking root)

$$= \lim_{x \rightarrow 0} [(1 + \sin 2x)^{1/\sin 2x}]^{\sin 2x/2x} = e$$

41. Ans. 1/4

(Hint. Limit $= \lim_{x \rightarrow \infty} \left(\frac{1 + (2/x) - (1/x^2)}{2 - (3/x) - (2/x^2)} \right)^{2 + (1/x) - (1/x)}$
 $= (1/2)^2 = 1/4$

42. Not continuous

(Hint : R.H.L. $= a \neq a - 1 =$ L.H.L.)

43. Ans : -1/2

(Hint : use $\tan^{-1} A - \tan^{-1} B$ formula, then put $1/x = t$,

$\therefore \pi/4 = \tan^{-1} 1$)

44. Ans (i) 0 (ii) b

(Hint: (i) Limit is of the form $0 \sin(\infty)$

$= 0 \times$ (Number between -1 and 1) $= 0$

(ii) Limit $= \lim_{x \rightarrow \infty} \frac{\sin(b/a^x)}{b/a^x}$. $b = b \times 1 = b$

45. Ans $|x|/x$

(Hint : Note that the limit is $= \frac{d}{dx} |x|$

It is easily seen that $\frac{d}{dx} |x| = \frac{|x|}{x}$

46. Ans : $\sqrt{ab}$

(Hint : You can use $x = e^{\log x}$ as in Q 18)

47. Ans : 5

(Hint : Limit $= \lim_{n \rightarrow \infty} \exp. [(1/n \log (3^n + 5^n))$

∞/∞ form. Use L-Hospital rule

$$= \lim_{n \rightarrow \infty} \left[\frac{3^n \log 3 + 5^n \log 5}{3^n + 5^n} \right]$$

$$= \lim_{n \rightarrow \infty} \exp. \left[\frac{(3/5)^n \log 3 + \log 5}{(3/5)^n + 1} \right]$$

$$= \lim_{n \rightarrow \infty} \exp. (\log 5) = 5$$

48. Ans : $-\infty$

(Hint : Given limit $= \lim_{n \rightarrow \infty} \frac{3x + (2/x^2)}{\sqrt{1 - (2/x^4)}}$
 $= \lim_{n \rightarrow \infty} \frac{(3x) + 0}{1} = -\infty$)

49. Ans: e^2

(Hint : See Q 46)

50. Ans: -2

(Hint : 0/0 form. Use L.H. rule)

51. Ans : me^{ma}

52. Ans : 0

(Hint: Apply L.H. rule n times: $= \lim_{x \rightarrow \infty} \frac{(n!) \cdot n^n}{e^{x/n}} = 0$)

53. Ans : $x \in \mathbb{R} \sim \{x : x \in \mathbb{I}\}$

i.e. all real x such that x is not an integer.

(Hint : See graph of $\{x\}$ or test R.H.L and L.H.L. at any integral value of x . R.H.L $\neq$ L.H.L)

54. Ans : $2 \tan a \sec^2 a$

55. Ans : $x \in \mathbb{R} \sim \{x : x \in \mathbb{I}\}$ (Hint : See Q 53)

56. Ans : Continuous

(Hint : In $f(x)$; divide above and below by $2^{1/x}$)

57. Ans : $x \in \mathbb{R}$

58. Ans : ∞

(Hint : Limit $= \lim x^{12} (6 - (8/x^5) + (9/x^{11}) + (1/x^{12}))$
 $= \infty \cdot 6 = \infty$

59. Ans : $-1/\pi$

60. Ans : ∞

(Hint : See Q 65)

61. Ans : $f(a)$

(Hint : L.H. rule applies, since as $x \rightarrow a^+$, the integral reduced to $\int_a^x f(t)dt$ which equals 0. $\therefore$ 0/0 form

$$\begin{aligned}\therefore \lim_{x \rightarrow a^+} \frac{1}{x-a} \int_a^x f(t) dt \\&= \lim_{x \rightarrow a^+} f(x) \quad (\because d/dx \int_a^x f(t) dt = f(x)) = f(a) \\&= f(a)\end{aligned}$$

62. Ans : ∞ (Hint : Apply L.H. rule 10 times)

63. Ans : $1/3$

64. Ans : e^3

(Hint : Put $x + 4 = t$: Limit $= \lim_{t \rightarrow \infty} \left(\frac{t+2}{t-3} \right)^t$

$$= \lim_{t \rightarrow \infty} \left(\frac{1 + (2/t)}{1 - (3/t)} \right)^t = e^2/e^3 = e^{-1}$$

65. Ans: not continuous

(Hint : L.H.L. = $-1 \neq$ R.H.L. = $1 \neq f(0) = 0$)

66. Ans : $-\infty$ (Hint : See Q 58)

67. Ans : 0

(Hint : Given limit

$$\lim_{x \rightarrow 0} \left[\left(1 + \frac{1}{n} \right)^n - 1 - \frac{1}{n} \right]^n = (e - 1)^n = 0$$

$\therefore x \rightarrow 0 \forall -1 < x < 1$

68. Ans : $8f'(1)$

(Hint : since $f(1) = 4$; at $x = 1$, the integral reduces to $\int_4^x 2t dt$ which is zero. The denominator is also 0. Thus it is 0/0 form and L.H. rule applies

$$\begin{aligned}\text{Limit} &= \lim_{x \rightarrow 1} \frac{d/dx \int_4^x 2t dt}{1} \\&= \lim_{x \rightarrow 1} \{ d/dx [f(x) + c] - d/dx [4^2 + c] \} \\&= \lim_{x \rightarrow 1} 2f(x) f'(x) = 2 \times 4 \times f'(1) = 8f'(1)\end{aligned}$$

69. Ans (iii)

70. Ans : 1

(Hint: Limit $= \lim_{x \rightarrow 0} \exp. \left(\frac{\log \sin x / x}{x} \right)$

0/0 form. Apply L.H. rule

$$= \lim_{x \rightarrow 0} \exp. (\cot x - (1/x)) \quad \text{Now see Q7)}$$

71. Ans : (b), (c), (d)

(Hint : $\tan x$ is discontinuous at $\pi/2$)

72. Ans : Limit does not exist

(hint : Limit $= \lim_{x \rightarrow 1} \frac{|x^2 - 1|}{x^2 - 1} \times \frac{2x}{x-1}$

$$\therefore \frac{d}{dx} |x| = \frac{|x|}{x} \quad \text{L.H.L.} = \lim_{x \rightarrow 1^-} -1 \times \frac{2x}{1-x} = -\infty$$

$$\text{R.H.L.} = \lim_{x \rightarrow 1^+} 1 \times \frac{2x}{x-1} = \infty$$

73. Ans : $a = 1, b = 2$

(Hint : Divide above and below by e^{2h} for R.H.L. and evaluate directly for L.H.L.)

74. Ans : 5 (Hint : Apply LH rule)

75. Ans : $\{n : n \in \mathbb{I}\} \cup \{n < 0\}$

76. Ans : $2/\pi$

(Hint : Put $x = 1 + h$; $x \rightarrow 1 \Rightarrow h \in 0$)

77. Ans : 1

(Hint : Limit $= \lim_{x \rightarrow 0} [\tan((\pi/4) + x)]^{\sin(1/x)}$

$$= \lim_{x \rightarrow 0} \left\{ \left(\frac{1 + \tan x}{1 - \tan x} \right)^{1/\tan x} \right\}^{\sin(1/x)} = (e/e^{-1})^0 = 1$$

78. Ans $\sqrt{a}$

79. Ans $c = 4, d = -1$

80. Ans $\sqrt{e}$ (Hint : Use $x = e^{\log x}$ and L.H. rule)

81. Ans = 0

(Hint : Use $\cos A - \cos B$, rationalize and you get the form $0 \sin(\infty) = 0$)

82. Ans : $-\pi^2$

Subjective Problems

83. Continuity can easily be shown at $x = 0$

Left hand derivative (L.H.D) $= \lim_{h \rightarrow 0} \frac{f(x_0 - h) - f(x_0)}{-h}$

$$= \lim_{h \rightarrow 0} \frac{(-h)^2 \sin(-1/h) - 0}{-h} = \lim_{h \rightarrow 0} h \sin(1/h) = 0$$

R.H.D. $= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$

$$= \lim_{h \rightarrow 0} \frac{h^2 \sin(1/h) - 0}{h} = 0$$

$\therefore$ Derivable at $x = 0$

$$\begin{aligned}
 84. \lim_{x \rightarrow 0} \frac{\cos(\sin x) - \cos x}{x^4} \\
 = \lim_{x \rightarrow 0} \frac{-2\sin(\sin x + (x/2)) \sin(\sin x - (x/2))}{x^4} \\
 = \lim_{x \rightarrow 0} -\frac{1}{2x^4} \cdot (\sin^2 x - x^2) \\
 0/0 \text{ form : apply L.H. rule 4 times} \\
 = 1/6
 \end{aligned}$$

$$\begin{aligned}
 85. \text{ Note that } 1 + x &= \frac{1 - x^2}{1 - x} \\
 (1 + x)(1 + x^2) &= \frac{1 - x^4}{1 - x} \\
 (1 + x)(1 + x^2)(1 + x^4) &= \frac{1 - x^8}{1 - x} \\
 \vdots \text{ Add all such terms till } n: \\
 (1 + x)(1 + x^2)(1 + x^4) \dots (1 + x^{2^n}) &= \frac{1 - x^{2^{n+1}}}{1 - x} \\
 \text{applying limit as } n \rightarrow \infty \text{ on both sides;} \\
 \lim_{n \rightarrow \infty} [(1 + x)(1 + x^2) \dots (1 + x^{2^n})] \\
 = \lim_{n \rightarrow \infty} \frac{1 - x^{2^{n+1}}}{1 - x} = \frac{1}{1 - x} \text{ Proved}
 \end{aligned}$$

$$\begin{aligned}
 86. \text{ Let } f(x) &= \cos x - x^n \\
 \text{Now, } f &\text{ is continuous on } [0, \pi/2] \\
 f(0) &= 1 \quad f(\pi/2) = -1 \\
 f(x) &= -(\sin x + nx^{n-1}) \\
 \text{which is negative in } &[0, \pi/2] \\
 \therefore f &\text{ is monotonically decreasing} \\
 \Rightarrow \text{There is a value of } f(x) &\text{ in } [0, \pi/2] \text{ which equals } 0, \\
 \Rightarrow \cos x - x^n &= 0 \\
 \text{or } \cos x = x^n &\text{ for that value. Proved}
 \end{aligned}$$

$$\begin{aligned}
 87. f(x) \text{ is differentiable at } x = 0 \text{ for } m \geq 2 \text{ and } f'(x) \\
 \text{is continuous for } m \geq 3. \\
 \therefore m \geq 3
 \end{aligned}$$

$$\begin{aligned}
 88. \text{ R.H.L} &= \lim_{x \rightarrow 0} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}} \\
 \text{Put } x &= 0 + h, x \rightarrow 0^+ \Rightarrow h \rightarrow 0 \\
 &= \lim_{h \rightarrow 0} \frac{h(3 + 4e^{1/h})}{2e^{1/h} - 1} = 0(-3) = 0 \\
 \text{L.H.L} &= \lim_{x \rightarrow 0} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}} \quad \text{Put } x = 0 - h \\
 &= \lim_{h \rightarrow 0} \frac{-h(3e^{-1/h} + 4)}{2 - 3e^{-1/h}} = -0(2) = 0 \\
 \therefore \text{L.H. L} &= \text{R.H. L} = f(0) \\
 \therefore f &\text{ is continuous and limit exists at } x = 0 \\
 \text{R.H.D} &= \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h}
 \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{h(3e^{1/h} + 4)}{h(2 - e^{1/h})} = \lim_{h \rightarrow 0} \frac{(3 + 4e^{1/h})}{(2e^{1/h} - 1)} = -3$$

$$\begin{aligned}
 \text{L.H.D} &= \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{-h(3e^{-1/h} + 4)}{h(2 - e^{-1/h})} = 4/2 = 2 \\
 \therefore \text{Derivative does not exist at } x &= 0
 \end{aligned}$$

$$\begin{aligned}
 89. \text{ Draw graph yourself} \\
 \text{Clearly, L.H.L} &= \text{R.H.L} = f(1) \text{ at } x = 1 \\
 \text{L.H.D} &= 3\log 3 \neq \text{R.H.D} = -1 \\
 \therefore \text{The function is not differentiable}
 \end{aligned}$$

$$\begin{aligned}
 90. \text{ L.H.L} &= \lim_{x \rightarrow 0^+} x \cdot \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \\
 \text{Put } x &= 0 - h, x \rightarrow 0 \Rightarrow h \rightarrow 0 \\
 &= \lim_{h \rightarrow 0} -h \cdot \frac{e^{-1/h} - e^{1/h}}{e^{-1/h} + e^{1/h}} = \lim_{h \rightarrow 0} -h \cdot \frac{e^{-2/h} - 1}{e^{-2/h} + 1} = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{Similarly, R.H.L} &= 0 \\
 \therefore \text{L.H.L} &= \text{R.H.L} = f(0) \\
 \text{The function is continuous at } x &= 0
 \end{aligned}$$

$$91. \text{ Draw the graph yourself}$$

$$\begin{aligned}
 f(x) &= \begin{cases} x^3, & |x| < 1 \\ x, & |x| \geq 1 \end{cases} \\
 \text{(Taking square root)} \\
 &= \begin{cases} x^3 - 1 & -1 < x < 1 \\ x & -1 \geq x \geq 1 \end{cases} \\
 \lim_{x \rightarrow 1} x^3 = 1 = f(1) = \lim_{x \rightarrow 1} x
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{The function is continuous at } x &= 1 \\
 f(1^+) &= \lim_{h \rightarrow 0} \frac{f(1 + h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{(1 + h) - 1}{h} = 1 \\
 f(1^-) &= \lim_{h \rightarrow 0} \frac{f(1 - h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{(1 - h)^3 - 1}{-h} = -3 \\
 \therefore \text{Not differentiable at } x &= 1
 \end{aligned}$$

$$92. \text{ Do yourself}$$

$$93. (f \circ g)(x) = \sqrt{\sin x - 1}$$

$$(g \circ f)(x) = \sin \sqrt{x - 1}$$

$$\text{Differentiability of } (g \circ f)(x) :$$

$$\begin{aligned}
 f(1^+) &= \lim_{h \rightarrow 0} \frac{f(1 + h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{\sin \sqrt{h} - 0}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin \sqrt{h}}{\sqrt{h}} \cdot \frac{1}{\sqrt{h}} = \infty \\
 f(1^-) &= \lim_{h \rightarrow 0} \frac{f(1 - h) - f(1)}{-h}
 \end{aligned}$$

$$= \lim_{h \rightarrow 0} \sin \frac{\sqrt{-h} - 0}{-h} = -\infty$$

∴ Derivative does not exist at $x = 1$

$$94. f(x) = \begin{cases} -x & -1 \leq x \leq -2 \\ -x & 0 \leq x < -1 \\ -x & 1 \leq x < 0 \\ x-2 & 2 \leq x < 1 \\ x-2 & 3 \leq x < 2 \end{cases}$$

$$g(x) = \begin{cases} x+3 & -1 \leq x \leq -2 \\ 1-x & 0 \leq x < -1 \\ 1-x & 1 \leq x < 0 \\ 1-x & 2 \leq x < 1 \\ 1-d & 3 \leq x < 2 \end{cases}$$

Now, find $(f \circ g)(x)$ and $(g \circ f)(x)$ for the given intervals

$f \circ g$ is continuous at $x = -1$

$g \circ f$ is not differentiable at $x = -1$

$$95. f(x) = \frac{2\cos x^2 - \sin x^2}{x^2}$$

$$= 2\cos x^2 - \frac{\sin x^2}{x^2}, x \neq 0$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} 2\cos x^2 - \frac{\sin x^2}{x^2}$$

$$= 2 - 1 = 1$$

$$= \lim_{x \rightarrow 0} 2\cos x^2 - \frac{\sin x^2}{x^2} = \lim_{x \rightarrow 0} f(x)$$

$$96. f(1) = c = c(1)$$

∴ The result is true for $x = 1$

Let it be true for $x = m$

Then; $f(m) = cm$

Let us prove it for $x = m + 1$ i.e.

$$f(m+1) = c(m+1)$$

$$\text{L.H.S} = f(m+1) = f(m) + f(1) \quad (\text{By given condition})$$

$$= cm + c = c(m+1) = \text{R.H.S}$$

So, the result stands true for $m = 1, 2, 3, 4, \dots$

$$97. \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{\sqrt[3]{x} - \sqrt[3]{a}} = \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{x - a}{\sqrt[3]{x} - \sqrt[3]{a}}$$

$$= \frac{1}{2} a^{1/2} \times (1/3 a^{1/3})^{-1} = \frac{3}{2} a^{1/6}$$

∴ $k = \frac{3}{2} a^{1/6}$ for the function to be continuous at $x = a$

98. Do yourself

(Hint : let $g(x) = ax + b$ $g(-1) = f(-1) = b - a$

Find $f'(x)$; put $x = 1$ and equate it to $b - a$. Then, find R.H.L. and L.H.L of $f(x)$ and equate the two to get value of b)

$$99. \text{ If } f(0) \text{ exists; } f(0^+) = f(0^-)$$

$$\therefore \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{f(-h) - f(0)}{-h}$$

f is even; $f(-h) = f(h)$ Now: L.H.S = - R.H.S

So, L.H.S = 0

$$\Rightarrow f'(0) = 0$$

$$100. \lim_{x \rightarrow 0} \{1 + |\sin x|\}^{\frac{1}{\sin x}} = e^1$$

$$\lim_{x \rightarrow 0^+} e^{\frac{\ln(1 + |\sin x|)}{\sin x}} = \lim_{x \rightarrow 0^+} \exp \left(\frac{\tan 2x}{2x} \times \frac{3x}{\tan 3x} \times \frac{2}{3} \right)$$

$$= e^{2/3} \therefore \text{L.H.L} = \text{R.H.L} = f(0);$$

$$a = 2/3; b = e^{2/3}$$

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INTEGRATION (AREA)

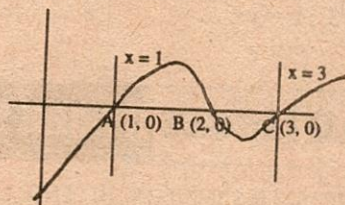
— Dr. Md. Motiur Rahman
Darbhanga, Bihar

1. Find the area bounded by the curve $y = (x-1)(x-2)(x-3)$ lying between the ordinate $x=1$ and $x=3$

Soln.

Required area

$$\begin{aligned}
 &= \int_1^3 y \, dx \\
 &= \int_1^3 |(x-1)(x-2)(x-3)| \, dx \\
 &= \int_1^2 |(x-1)(x-2)(x-3)| \, dx + \int_2^3 |(x-1)(x-2)(x-3)| \, dx \\
 &= \int_1^2 (x-1)(x-2)(x-3) \, dx - \int_2^3 (x-1)(x-2)(x-3) \, dx \\
 &= \int_1^2 (x^3 - 6x^2 + 11x - 6) \, dx - \int_2^3 (x^3 - 6x^2 + 11x - 6) \, dx \\
 &= \left[\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x \right]_1^2 - \left[\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x \right]_2^3 \\
 &= \left(-2 + \frac{9}{4} \right) - \left(\frac{-9}{4} + 2 \right) = \frac{1}{2}
 \end{aligned}$$



2. The area bounded by the curve $y = f(x)$, x axis and the ordinates $x=1$ and $x=b$ is $(b-1)\sin(3b+4)$. Find $f(x)$

Soln.

$$\text{Given, } (b-1)\sin(3b+4) = \int_1^b y \, dx = \int_1^b f(x) \, dx$$

Differentiating w.r.t. b we get

$$\begin{aligned}
 \sin(3b+4) + 3(b-1)\cos(3b+4) &= f(b) \\
 \therefore f(x) &= \sin(3x+4) + 3(x-1)\cos(3x+4)
 \end{aligned}$$

3. Sketch the region bounded by the curve $y = \sqrt{5-x^2}$ and $y = |x-1|$ and find its area

Soln.

$$y = |x-1|$$

$$\therefore y = x-1, x \geq 1 \quad \dots\dots(1)$$

$$y = -x+1, x < 1 \quad \dots\dots(2)$$

represents two straight lines

$$\text{Also } y = \sqrt{5-x^2} \quad \dots\dots(3)$$

represents the upper half of the circle $x^2 + y^2 = 5$

Solving (1) and (3) we have,

$$\sqrt{5-x^2} = x-1 \Rightarrow 5-x^2 = x^2-2x+1$$

$$\text{or } 2x^2-2x-4=0 \text{ or } x^2-x-2=0$$

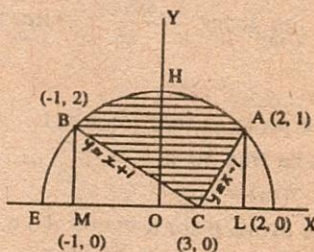
$$\text{or } x^2-2x+x-2=0 \text{ or } x(x-2)+1(x-2)=0$$

$$\Rightarrow x = -1, 2$$

Solving (2) and (3) we get $x = -1$

The required area is the shaded region

$$\begin{aligned}
 &= \int_{-1}^2 \sqrt{5-x^2} \, dx \\
 &\quad - \int_{-1}^1 (-x+1) \, dx \\
 &\quad - \int_1^2 (x-1) \, dx \\
 &= \left[\frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^2 - \left[-\frac{x^2}{2} + x \right]_{-1}^1
 \end{aligned}$$



$$\begin{aligned}
 &= \left[1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} \right] - \left[-1 + \frac{5}{2} \sin^{-1} \left(\frac{-1}{\sqrt{5}} \right) \right] \\
 &= \frac{5}{2} \left(\sin^{-1} \frac{2}{\sqrt{5}} + \sin^{-1} \frac{1}{\sqrt{5}} \right) - \frac{1}{2} \\
 &= \frac{5}{2} \sin^{-1} \left(\frac{2}{\sqrt{5}} \sqrt{1-\frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1-\frac{4}{5}} \right) - \frac{1}{2} \\
 &= \frac{5}{2} \sin^{-1} 1 - \frac{1}{2} = \frac{5\pi}{4} - \frac{1}{2}
 \end{aligned}$$

4. Find the area bounded by the curve $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$, above x axis.

Soln.

$x^2 + y^2 = 25$ (1) represents a circle whose centre is (0, 0) and radius 5

Also $4y = |4 - x^2|$

$4y = 4 - x^2$ if $-2 \leq x \leq 2$ (2)

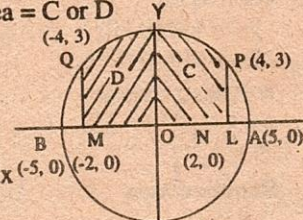
$= -4 + x^2$ if $x > 2$ or $x < -2$ (3)

or $x^2 = -4(y - 1)$ for $-2 \leq x \leq 2$

which is a parabola whose vertex is (0, 1)

Similarly $x^2 = 4(y + 1)$ for $x > 2$ or $x < -2$ represents a parabola whose vertex is (0, -1)

Point of intersection of (2) and (3) are (2, 0) and (-2, 0). Point of intersection of (1) and (3) are (4, 3) and (-4, 3) $\Rightarrow$ Required area = C or D

$$= \int_0^4 \sqrt{25 - x^2} dx - \int_0^2 \frac{4 - x^2}{4} dx - \int_2^4 \frac{x^2 - 4}{4} dx$$


$$= \left[\frac{x}{2} \sqrt{25 - x^2} + \frac{25}{2} \sin^{-1} \frac{x}{5} \right]_0^4 - \frac{1}{4} \left[4x - \frac{x^3}{3} \right]_0^2$$

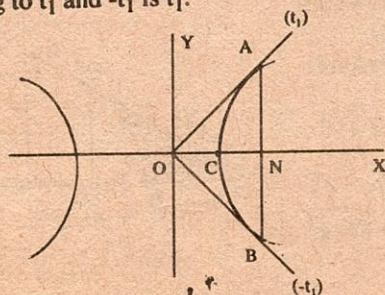
$$- \frac{1}{4} \left[\frac{x^3}{3} - 4x \right]_2^4 = 2 + \frac{25}{2} \sin^{-1} \frac{4}{5}$$

5. For any real t , $x = \frac{1}{2}(e^t + e^{-t})$,

$y = \frac{1}{2}(e^t - e^{-t})$ is a point on the hyperbola

$x^2 - y^2 = 1$. Show that the area bounded by the hyperbola and the lines joining its centre to the points corresponding to t_1 and $-t_1$ is t_1 .

Soln.



Let A and B be two points corresponding to t_1 and

$-t_1$, then we have

$$A = \left(\frac{e^{t_1} + e^{-t_1}}{2}, \frac{e^{t_1} - e^{-t_1}}{2} \right)$$

$$\text{and } B = \left(\frac{e^{-t_1} + e^{t_1}}{2}, \frac{e^{-t_1} - e^{t_1}}{2} \right)$$

Required area = area OACBO

$$= \Delta AOB - 2 \text{ area OCN}$$

$$\text{Now } \Delta AOB = \frac{1}{2} ABON$$

$$= \frac{1}{2} (e^{t_1} - e^{-t_1}) \left(\frac{e^{t_1} + e^{-t_1}}{2} \right)$$

$$= \frac{1}{4} (e^{2t_1} - e^{-2t_1})$$

$$2 (\text{Area ACN}) = 2 \int_0^{t_1} y dx$$

$$= 2 \int_0^{t_1} \left(\frac{e^t - e^{-t}}{2} \right) \left(\frac{e^t + e^{-t}}{2} \right) dt$$

$$= \frac{1}{2} \int_0^{t_1} (e^{2t} + e^{-2t} - 2) dt$$

$$= \frac{1}{4} (e^{2t_1} - e^{-2t_1}) - t_1$$

Hence required area = t_1

6. Determine the region bounded by

$$x^2 + y^2 - 2x \leq 0,$$

$$x + y \leq 1, y \geq 0$$

Soln.

The given equations are

$$x^2 + y^2 - 2x \leq 0 \text{(1)}$$

$$x + y \leq 1 \text{(2)}$$

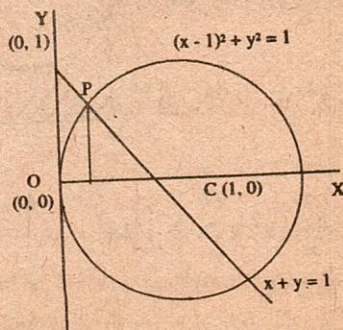
$$y \geq 0 \text{(3)}$$

(1) represents the interior and boundary of the circle

$$(x - 1)^2 + y^2 = 1$$

(2) represents the left half of the plane determined by the line $x + y = 1$, including the line.

(3) represents



plane w.r.t. x axis including x axis.

The circle and the line

meet at $\left(1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$\text{Required area} = \int_0^{1 - \frac{1}{\sqrt{2}}} \sqrt{1 - (x-1)^2} dx - \int_0^{1 - \frac{1}{\sqrt{2}}} (1-x) dx$$

$$= \left[\frac{x-1}{2} \sqrt{1 - (x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{1 - \frac{1}{\sqrt{2}}} - \left[x - \frac{x^2}{2} \right]_0^{1 - \frac{1}{\sqrt{2}}}$$

$$= \frac{\pi}{8} - \frac{1}{2}$$

7. Find the area bounded by the curve $y = x^2 - 2x + 2$, the line tangent to it at the point (3, 5) and the axes of coordinates

Soln.

The given equation is

$$y = x^2 - 2x + 2 \Rightarrow y - 1 = (x - 1)^2$$

Which is a parabola whose vertex a is (1, 1)

$$\frac{dy}{dx} = 2x - 2$$

$$\text{At } (3, 5) \frac{dy}{dx} = 4$$

Hence equation of the tangent at (3, 5) is

$$y - 5 = 4(x - 3) \Rightarrow 4x - y - 7 = 0$$

Required area =

$$\begin{aligned} & \int_0^3 (x^2 - 2x + 2) dx - \int_0^3 (4x - 7) dx \\ &= \left[\frac{x^3}{3} - x^2 + 2x \right]_0^3 - \left[2x^2 - 7x \right]_0^3 = \frac{23}{8} \end{aligned}$$

8. Find the area bounded by the curve $x^2 + 2x + y - 3 = 0$, x axis and the tangent at the point where it meets y axis.

Soln.

The given equation is

$$x^2 + 2x + y - 3 = 0 \quad \dots\dots\dots(1)$$

$$(x + 1)^2 = 4 - y$$

Which is a parabola

$$x^2 + 2x + y - 3 = 0 \Rightarrow 2x + 2 + \frac{dy}{dx} = 0$$

(1) meets y axis at (0, 3)

$$\text{At } (0, 3) \frac{dy}{dx} = -2$$

Equation of tangent

at (0, 3) is

$$y - 3 = -2(x - 0)$$

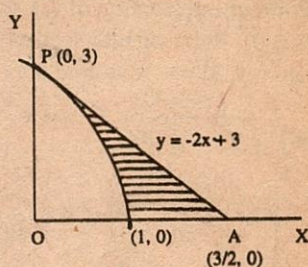
$$2x + y - 3 = 0$$

It meets x axis at

$$A\left(\frac{3}{2}, 0\right)$$

Required area

$$= \int_0^{3/2} [(-2x + 3) - (-x^2 - 2x + 3)] dx = \frac{7}{12}$$



9. Find the area bounded by the curve $y = \sqrt{8 - x^2}$, tangent to it at the point with abscissa $x = -2$ and the x axis.

Soln.

$$y = \sqrt{8 - x^2}$$

when $x = -2$, $y = 2$ so P is (-2, 2)

$$\frac{dy}{dx} = \left[-\frac{x}{\sqrt{8 - x^2}} \right] = 1$$

$$(-2, 2)$$

Equation of tangent at P(-2, 2) is

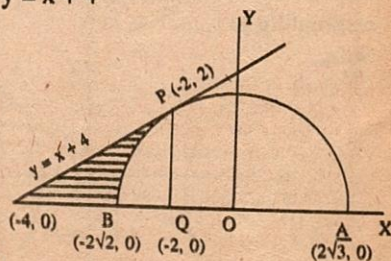
$$y - 2 = x + 2 \Rightarrow y = x + 4$$

It meets x axis

at (-4, 0)

Required area

$$\begin{aligned} &= \int_{-4}^{-2} (x + 4) dx - \int_{-2\sqrt{2}}^{-2} \sqrt{8 - x^2} dx \\ &= \left[\frac{x^2}{2} + 4x \right]_{-4}^{-2} - \left[\frac{x}{2} \sqrt{8 - x^2} + 4 \sin^{-1} \frac{x}{\sqrt{8}} \right]_{-2\sqrt{2}}^{-2} \end{aligned}$$



C.B.S.E. MED. ENT.

A Tradition of Success!
Our Student Bags

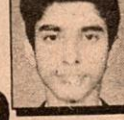
1st Position for the 6th Consecutive Year

TOPPERS

1991	1992	1993	1994	1995	1996
					
1 st ST PANKAJ SRIVASTAVA	1 st ST PANKAJ AMITE	1 st ST ROHTASHAV DHIR	1 st ST NEERAJ SINHA	1 st ST HEMANT GOYAL	1 st ST PRIYANK JAIN

MORE THAN 200 SELECTIONS IN 1996 ALONE

OUR MERIT RANK HOLDERS

					
1 st PRIYANK JAIN	2 nd HARVINDER SINGH ARORA	3 rd SINDHU CHANDRAN	4 th SHIKHA GOYAL	6 th MANPREET KAUR	9 th VIKAS DUDEJA

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OUR HEARTIEST CONGRATULATIONS TO ALL OF THEM!

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Bus Stand Rd. ★ BHATINDA Power House Road ★ BHILAI New Civic Centre ★ VALSAD Township ★ BHOPAL Mahiya Nagar ★ BHUBANESHWAR
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Kotla M. pur ★ Chawri Bazar ★ Madhuban Chowk ★ FARIDABAD Sector-16 ★ GHAZIABAD Ambedkar Road ★ GORAKHPUR Doudpur
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SAMBALPUR Computing House ★ SAHARANPUR Court Road ★ VARANASI Lahurabi ★ Brij Enclave

HOSTEL FACILITY & CORRESPONDENCE COURSES ALSO AVAILABLE

$$= \left[\frac{x^2}{2} + 4x \right]_{-4}^{-2} - \left[\frac{x}{2} \sqrt{8-x^2} + \frac{8}{2} \sin^{-1} \frac{x}{\sqrt{8}} \right]_{-4}^{-2}$$

$$= 4 - \pi$$

10. Find the area bounded by the parabola $y^2 = 2ax$ and a normal to it inclined at angle of 135° with the x axis.

Soln.

$$y^2 = 2ax \Rightarrow \frac{dy}{dx} = \frac{a}{y}$$

$$\Rightarrow -\frac{dx}{dy} = -\frac{y}{a} = \tan 135 = -1$$

$$\therefore y = a. \text{ Hence } x = \frac{a}{2}$$

Equation of normal at $\left(\frac{a}{2}, a\right)$ is

$$y - a = -\left(x - \frac{a}{2}\right) \text{ or } x + y - \frac{3a}{2} = 1$$

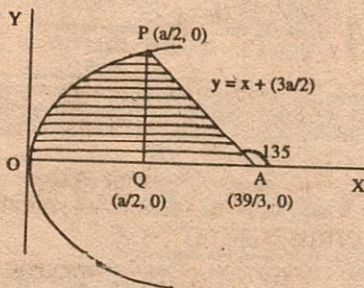
It meets x axis at $A\left(\frac{3a}{2}, 0\right)$

Required area

$$\int_0^{\frac{a}{2}} \sqrt{2ax} \, dx$$

$$+ \int_{\frac{a}{2}}^{\frac{3a}{2}} \left(-x + \frac{3a}{2}\right) dx$$

$$= \frac{16}{3} a^2$$



11. Find the area bounded by the parabola $y = 6 + 4x - x^2$ and the chord joining the points $(-2, -6)$ and $(4, 6)$

Soln.

The given equation is

$$y = 6 + 4x - x^2 \text{ or } y = 10 - (x - 2)^2$$

$$\text{or } (x - 2)^2 = -(y - 10)$$

Which is a parabola whose vertex is $(2, 10)$ and the axis $x = 2$

It cuts x axis and y axis at $(0, 6)$ and $(2 \pm \sqrt{10}, 0)$

The equation of the chord joining the two given points $(-2, -6)$ and $(4, 6)$ is

$$\frac{y+6}{-6-6} = \frac{x+2}{-2-4} \text{ or } 2x - y - 2 = 0$$

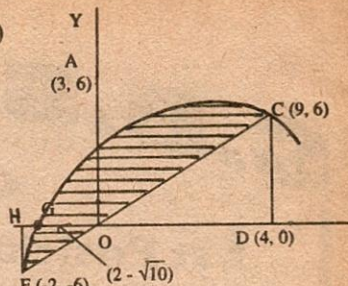
It meets x axis at $(1, 0)$

Required area

$$= \left| \int_{-2}^4 (6 + 4x - x^2) dx \right|$$

$$- \left| \int_{-2}^4 [2(x+2) - 6] dx \right|$$

$$+ \left| \int_{-2}^1 [2(x+2) - 6] dx \right| - \left| \int_{-2}^{2-\sqrt{10}} (6 + 4x - x^2) dx \right| = 36$$



12. The line $y = mx$ bisects the area enclosed by the line $x = 0$, $y = 0$, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$.

Find the value of m .

Soln.

The given equation is

$$y = 1 + 4x - x^2$$

or $(x - 2)^2 = -(y - 5)$ which is a parabola with vertex at $(2, 5)$ and the line $x = 2$ is its axis

Area OABC

$$\int_0^{3/2} y \, dx$$

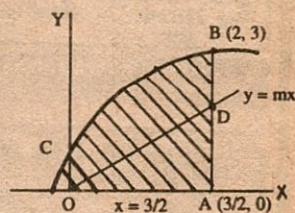
$$= \int_0^{3/2} (1 + 4x - x^2) dx$$

$$= \left[x + 2x^2 - \frac{x^3}{3} \right]_0^{3/2} = \frac{39}{8}$$

$$\text{Area OAD} = \int_0^{3/2} y \, dx = \int_0^{3/2} mx \, dx$$

$$= \frac{m}{2} \left[x^2 \right]_0^{3/2} = \frac{9m}{8}$$

$$\text{Given } \frac{9m}{8} = \frac{39}{16} \Rightarrow m = \frac{13}{6}$$



13. Find the area enclosed by the curves $x^2 + y^2 = 4$, $x^2 + y^2 - 6x + 8 = 0$ and a common tangent to the curves.

Soln.

Let the tangent at (x_1, y_1) to the circle $x^2 + y^2 = 4$ also touch other circle

$\therefore xx_1 + yy_1 = 4$ touches the circle $x^2 + y^2 - 6x + 8 = 0$ whose centre is (3, 0) and radius 1.

$$\therefore \left| \frac{3x_1 + 0 \cdot y_1 - 4}{\sqrt{x_1^2 + y_1^2}} \right| = 1 \Rightarrow \left| \frac{3x_1 - 4}{2} \right| = 1$$

Since $x_1^2 + y_1^2 = 4$

$$\therefore x_1 = 3, \frac{2}{3}$$

Hence the point of contact P of the common tangent

$$= \left(\frac{2}{3}, \frac{4\sqrt{2}}{3} \right) \text{ on the first circle.}$$

The equation of common tangent is

$$x \cdot \frac{2}{3} + \frac{y \cdot 4\sqrt{2}}{3} = 4$$

$$\text{ie. } x + 2\sqrt{2}y = 6$$

Let the point of contact on the second circle be

(x_2, y_2)

Then $xx_2 + yy_2 - 3(x + x_2) + 8 = 0$ touches the circle $x^2 + y^2 = 4$

$$\therefore \frac{0(x_2 - x_3) + 0 \cdot y_2 + 8 - 3x_2}{\sqrt{(x_2 - 3)^2 + y_2^2}} = 2$$

$$\text{or } \left| \frac{8 - 3x_2}{\sqrt{x_2^2 + y_2^2 - 6x_2 + 9}} \right| = 2$$

$$\text{or } \left| \frac{8 - 3x_2}{1} \right| = 2 \Rightarrow x_2 = \frac{10}{3}, 3$$

When $x = \frac{10}{3}$, $x_2^2 + y_2^2 - 6x_2 + 8 = 0$ gives

$$y^2 = \pm \frac{2\sqrt{2}}{3}$$

Hence the point of contact

$$Q = \left(\frac{10}{3}, \frac{2\sqrt{2}}{3} \right)$$

Required area

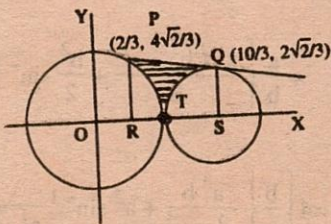
= area (PQSR)

- area (PRTP)

- area (QTSQ)

$$= \int_{2/3}^{10/3} \frac{6-x}{2\sqrt{2}} dx - \int_{2/3}^2 \sqrt{4-x^2} dx - \int_2^{10/3} \sqrt{6x-x^2-8} dx$$

$$= \frac{1}{2\sqrt{2}} \left[6x - \frac{x^2}{2} \right]_{2/3}^{10/3} - \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{2/3}^2$$



$$= \left[\frac{(x-3)\sqrt{1-(x-3)^2}}{2} + \frac{1}{2} \sin^{-1} \frac{x-3}{1} \right]_{2/3}^{10/3}$$

$$= 3\sqrt{2} + \frac{3}{2} \sin^{-1} \frac{1}{3} - \frac{5\pi}{4}$$

14. Find the area of the region

$\{(x, y) : x^2 \leq y \leq |x|\}$

Soln.

The equation $x^2 = y$ represents a parabola

The equation $y = |x|$ represents a pair of straight lines $y = x, x \geq 0$ and $y = -x$ for $x < 0$

The parabola $y = x^2$ and

the line $y = x$ intersect

at the point

$(0, 0)$ and $(1, 1)$

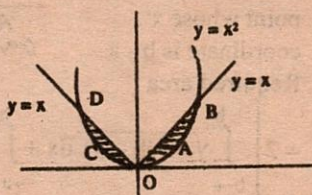
Required area

= area OAB

+ area OCD

= 2 area OAB

$$= 2 \left[\int_0^1 x dx - \int_0^1 x^2 dx \right] = 2 \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{3}$$



15. Find the area of the region

$\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

Soln.

The equation $y^2 = 4x$ represents a parabola and the

equation $x^2 + y^2 = \frac{9}{4}$ represents a circle

The two curves

intersect at the points

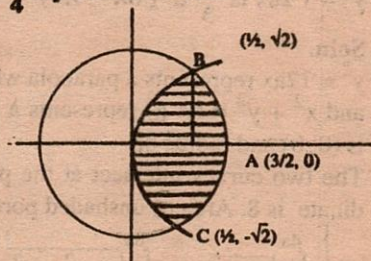
$\left(\frac{1}{2}, \sqrt{2} \right)$ and

$\left(\frac{1}{2}, -\sqrt{2} \right)$

Required area

$$= 2 \left[\int_0^{1/2} 2\sqrt{x} dx + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - x^2} dx \right]$$

$$= 2 \left[\frac{4}{3} \left\{ x^{3/2} \right\}_0^{1/2} + \left\{ \frac{9}{8} \sin^{-1} \frac{2x}{3} + \frac{x}{2} \sqrt{\frac{9}{4} - x^2} \right\}_{1/2}^{3/2} \right]$$



$$= 2 \left[\frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \frac{1}{3} - \frac{1}{4} \sqrt{2} \right]$$

$$= 2 \left[\frac{\sqrt{2}}{12} + \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \frac{1}{3} \right]$$

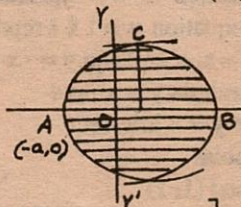
16. Show that the area included between the parabolas

$$y^2 = 4a(x + a) \text{ and } y^2 = 4b(b - x) \text{ is } \frac{8}{3} \sqrt{ab} (a + b)$$

Soln.

The two equations $y^2 = 40(x + a)$ and $y^2 = 4b(b - x)$ represent two parabolas whose vertex are $(-a, 0)$ and $(b, 0)$.

The two parabolas intersect at the point whose x coordinate is $b - a$



Required area

$$= 2 \left[\int_{b-a}^b \sqrt{4b(b-x)} dx + \int_{-a}^{b-a} \sqrt{4a(a+x)} dx \right]$$

$$= 2 \left[2\sqrt{b} \left\{ -\frac{2}{3} (b-x)^{3/2} \right\} + 2\sqrt{a} \left\{ \frac{2}{3} (a+x)^{3/2} \right\} \right]_{b-a}^b$$

$$= \frac{8}{3} \sqrt{ba}^{3/2} + \frac{8}{3} \sqrt{ab}^{3/2} = \frac{8}{3} \sqrt{ab} (a + b)$$

17. Show that the larger of the two areas into which the circle $x^2 + y^2 = 64a^2$ is divided by the parabola

$$y^2 = 12ax \text{ is } \frac{16}{3} a^2 (8\pi - \sqrt{3})$$

Soln.

$y^2 = 12ax$ represents a parabola whose vertex is $(0, 0)$ and $x^2 + y^2 = 64a^2$ represents a circle whose centre is $(0, 0)$ and radius $8a$

The two curves intersect at the point whose x coordinate is 8. Area of unshaded portion

$$= 2 \left[\int_0^{4a} \sqrt{12ax} dx + \int_{4a}^{8a} \sqrt{64a^2 - x^2} dx \right]$$

$$= 2 \left[2\sqrt{3a} \frac{2}{3} \left\{ x^{3/2} \right\}_0^{4a} + \left\{ \frac{x}{2} \sqrt{64a^2 - x^2} + \frac{1}{2} 64a^2 \sin^{-1} \left(\frac{x}{8a} \right) \right\}_{4a}^{8a} \right]$$

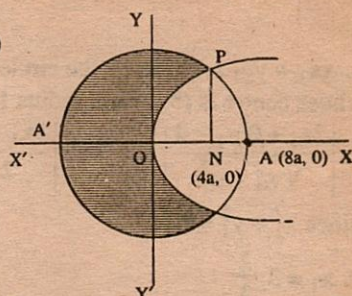
$$= \frac{16a^2}{3} (4\pi + \sqrt{3})$$

Hence required

$$\text{area} = 64\pi a^2 -$$

$$\frac{16a^2}{3} (4\pi + \sqrt{3})$$

$$= \frac{16a^2}{3} (8\pi - \sqrt{3})$$

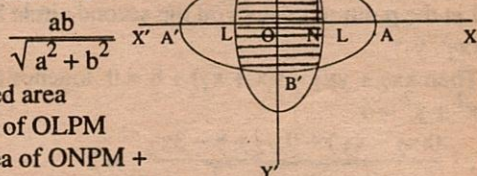


18. Show that the area between the ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \text{ is } 4ab \sin^{-1} \frac{b}{\sqrt{a^2 + b^2}}$$

Soln.

The two equations represent an ellipse. x coordinate of P =



Required area

$$= 4 \text{ area of OLPM}$$

$$= 4 [\text{area of ONPM} + \text{area of PNL}]$$

$$= 4 \left[\frac{ab}{\sqrt{a^2 + b^2}} \int_0^{\frac{b}{\sqrt{a^2 + b^2}}} \sqrt{a^2 - x^2} dx + \frac{a}{b} \int_{\frac{ab}{\sqrt{a^2 + b^2}}}^b \sqrt{b^2 - x^2} dx \right]$$

$$= 4 \frac{b}{a} \left\{ \frac{x \sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\}_0^{\frac{b}{\sqrt{a^2 + b^2}}} + \frac{a}{b} \left\{ \frac{x \sqrt{b^2 - x^2}}{2} + \frac{b^2}{2} \sin^{-1} \frac{x}{b} \right\}_{\frac{ab}{\sqrt{a^2 + b^2}}}^b$$

$$= 4 \left[\frac{b}{a} \left\{ \frac{a^3 b}{a^2 + b^2} + a^2 \sin^{-1} \frac{b}{\sqrt{a^2 + b^2}} \right\} + \frac{a}{b} \left\{ \frac{\pi b^2}{2} - \frac{ab^3}{a^2 + b^2} - b^2 \sin^{-1} \frac{a}{\sqrt{a^2 + b^2}} \right\} \right]$$

$$= 4ab \left[\sin^{-1} \frac{b}{\sqrt{a^2 + b^2}} + \frac{\pi}{2} - \sin^{-1} \frac{a}{\sqrt{a^2 + b^2}} \right]$$

$$= 2ab \left[\sin^{-1} \frac{b}{\sqrt{a^2 + b^2}} + \cos^{-1} \frac{a}{\sqrt{a^2 + b^2}} \right]$$

$$= 2ab \left[\sin^{-1} \frac{b}{\sqrt{a^2+b^2}} + \sin^{-1} \frac{b}{\sqrt{a^2+b^2}} \right]$$

$$= 4ab \sin^{-1} \frac{b}{\sqrt{a^2+b^2}}$$

19. Find the area of the curve

$$5x^2 + 6xy + 2y^2 + 7x + 6y + 6 = 0$$

Soln.

The given equation is

$$2y^2 + 6y(x+1) + (5x^2 + 7x + 6) = 0$$

This is a quadratic equation in y so it will have two roots. Let two roots be y_1 and y_2 .

$$y_1 + y_2 = -3(x+1)$$

$$y_1 y_2 = \frac{5x^2 + 7x + 6}{2}$$

$$\begin{aligned} \text{Now, } (y_1 - y_2)^2 &= (y_1 + y_2)^2 - 4y_1 y_2 \\ &= 9(x+1)^2 - (5x^2 + 7x + 6) \\ &= -x^2 + 4x - 3 \end{aligned}$$

$$y_1 - y_2 = \sqrt{4x - x^2 - 3}$$

The value of y is real when x lies between (1) and (3)

Hence the required area

$$\begin{aligned} &= \int_1^3 (y_1 - y_2) dx \\ &= \int_1^3 \sqrt{1 - (x-2)^2} dx \\ &= \frac{1}{2} \left[(x-2) \sqrt{1 - (x-2)^2} + \sin^{-1} (x-2) \right]_1^3 \\ &= \frac{1}{2} \left[\sin^{-1} 1 - \sin^{-1} (-1) \right] = \frac{\pi}{2} \end{aligned}$$

20. Find the area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$

Soln.

The given equations are

$$y = 2x - x^2 \quad \dots\dots(1)$$

$$\text{and } y = -x \quad \dots\dots(2)$$

(1) can be written as

$$x^2 - 2x + 1 = -y + 1 \text{ or } (x-1)^2 = -(y-1)$$

Which is a parabola whose vertex is (1, 1)

The point of intersection of (1) and (2) is (3, -3)

Required area

$$= \text{Area OAB} +$$

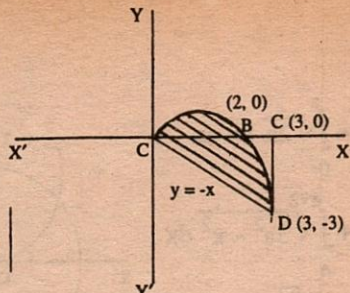
$$\Delta OCD - \text{area BCD}$$

$$= \int_0^2 (2x - x^2) dx$$

$$+ \frac{1}{2} \cdot 3 \cdot 3 - \left| \int_2^3 y dx \right|$$

$$= \left\{ x^2 - \frac{x^3}{3} \right\}_0^2 + \frac{9}{2} - \left\{ x^2 - \frac{x^3}{3} \right\}_2^3$$

$$= 4 - \frac{8}{3} + \frac{9}{2} - \left| 5 - \frac{19}{3} \right| = \frac{4}{3} + \frac{9}{2} - \frac{4}{3} = \frac{9}{2}$$



21. Compute the area of the figure bounded by the parabola $x = -2y^2$ and $x = 1 - 3y^2$

Soln.

The two equations $x = -2y^2$ and $x = 1 - 3y^2$ represent parabolas whose vertex are (0, 0) and (1, 0)

$$\text{Also } -2y^2 = 1 - 3y^2$$

$$\Rightarrow y^2 = 1$$

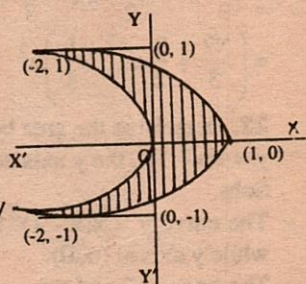
$$y = \pm 1$$

$$\text{Also } 1 - 3y^2 \geq -2y^2$$

$$\Rightarrow -1 \leq y \leq 1$$

Required area

$$\begin{aligned} &= \int_{-1}^1 [(1 - 3y^2) - (-2y^2)] dy \\ &= \int_{-1}^1 (1 - y^2) dy = \frac{4}{3} \end{aligned}$$



22. Find the area of the figure which lies in the first quadrant inside the circle $x^2 + y^2 = 3a^2$ and bounded by the parabolas $x^2 = 2ay$ and $y^2 = 2ax$ ($a > 0$)

Soln.

The given equations are

$$y^2 = 2ax \quad \dots\dots(1)$$

$$x^2 = 2ay \quad \dots\dots(2)$$

$$\text{and } x^2 + y^2 = 3a^2 \quad \dots\dots(3)$$

Equations (1) and (2) represent parabolas where (3) represents a circle x coordinate of point of intersection of (1) and (3), (2) and (3) are a , $a\sqrt{2}$

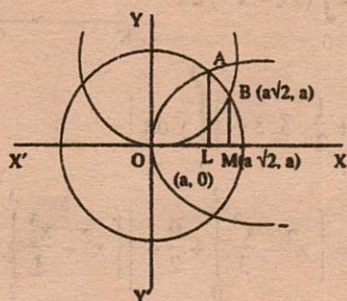
Required area

$$= \text{Area OAC}$$

$$+ \text{Area CABD}$$

$$- \text{Area OBD}$$

$$= \int_0^a \sqrt{2a} \sqrt{x} dx + \int_0^{a\sqrt{2}} \sqrt{3a^2 - x^2} dx - \int_0^{a\sqrt{2}} \frac{x^2}{2a} dx$$



$$= \left[\frac{2}{3} \sqrt{2a} x^{3/2} \right]_0^a + \left[\frac{x}{2} \sqrt{3a^2 - x^2} + \frac{3a^2}{2} \sin^{-1} \frac{x}{a\sqrt{2}} \right]_0^{a\sqrt{2}} - \left[\frac{1}{2a} \frac{x^3}{3} \right]_0^{a\sqrt{2}}$$

$$= \frac{2\sqrt{2}}{3} a^2 + \frac{3a^2}{2} \sin^{-1} \frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{3} a^2 = \left(\frac{\sqrt{2}}{3} + \frac{3}{2} \sin^{-1} \frac{1}{\sqrt{2}} \right) a^2$$

23. Determine the area bounded by the curve $y = x(x-1)^2$, the y axis and the line $y = 2$

Soln.

The curve $y = x(x-1)^2$ meets x axis at (0, 0), (1, 0) while y axis at (0, 0).

The line $y = 2$ and the curve $y = x(x-1)^2$ meet at (2, 2)

Required area

= area ODBC - (area OEA + area ABD)

$$= \int_0^2 2 dx - \int_0^2 x(x-1)^2 dx$$

$$= 4 - \int_0^2 (x^3 - 2x^2 + x) dx = 4 - \left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} \right]_0^2$$

$$\Rightarrow 4 - \frac{2}{3} = \frac{10}{3}$$

24. Find the area bounded by the curve $y = 2x^4 - x^2$, x axis and the two ordinates corresponding to the minima of the function

Soln.

$$\text{Here } y = 2x^4 - x^2$$

$$\frac{dy}{dx} = 8x^3 - 2x = 2x(4x^2 - 1)$$

For maxima or minima

$$\frac{dy}{dx} = 0 \quad x = 0, \pm \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 24x^2 - 2$$

$$\text{At } x = \pm \frac{1}{2}, \frac{d^2y}{dx^2} = 4 > 0$$

Thus y will be minimum at $x = \pm \frac{1}{2}$

$$\text{Required area} = \int_{-1/2}^{1/2} (2x^4 - x^2) dx$$

$$= 2 \int_0^{1/2} (2x^4 - x^2) dx \Rightarrow 2 \left[\frac{2x^5}{5} - \frac{x^3}{3} \right]_0^{1/2} = 2 \left[\frac{1}{80} - \frac{1}{24} \right]$$

$$\Rightarrow \frac{(3-10)^2}{240} = \frac{-7}{120} = \frac{7}{120} \text{ (in magnitude)}$$

25. Find the area of the region bounded between the two circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$

Soln.

The two equations $x^2 + y^2 = 1$ and

$(x-1)^2 + y^2 = 1$ represent circle whose centres are (0, 0) and (1, 0).

Their radii are 1.

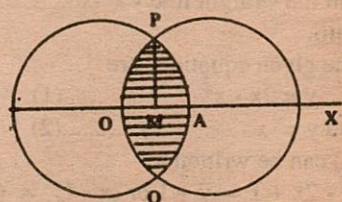
The coordinates

of P are

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

Required area

$$= 2 \left[\int_0^{1/2} \sqrt{1 - (x-1)^2} dx + \int_{1/2}^1 \sqrt{1 - x^2} dx \right]$$



Contd. on page No. 17

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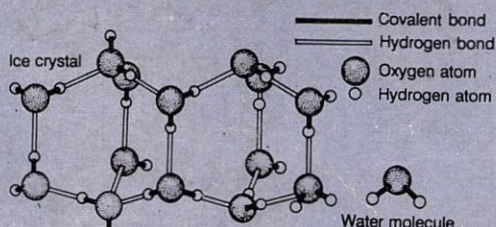
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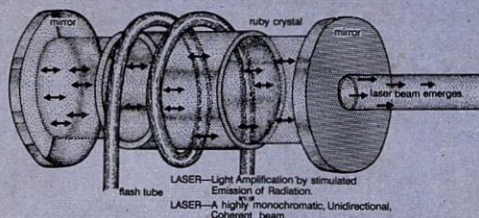
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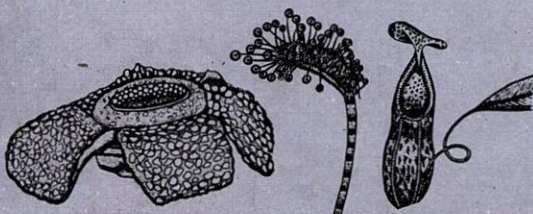


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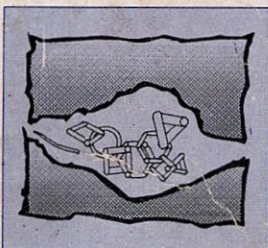
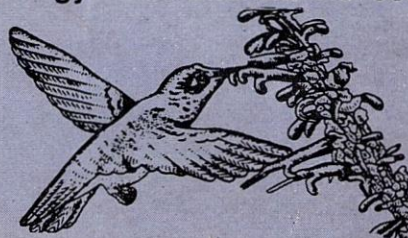
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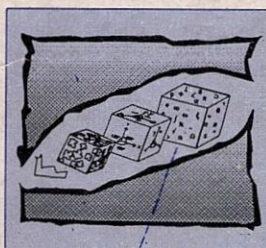
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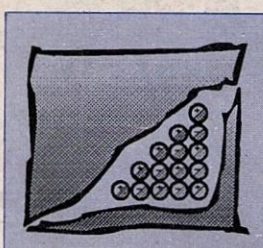
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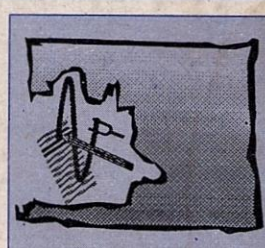
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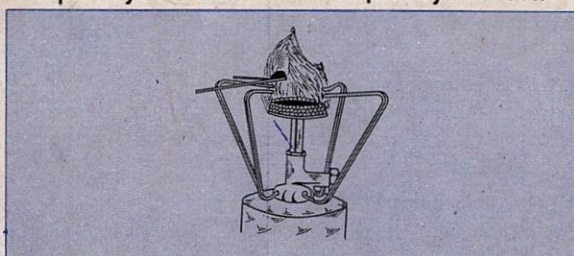
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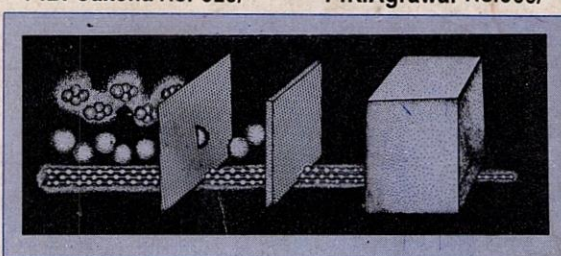
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